### Strain Measurements

#### Topics
- Stress and Strain
- Resistance Strain Gauges
- Strain Gauge Electrical Circuits
- Apparent Strain and Temperature Compensation
- Optical Strain Measuring Techniques

#### References

### Strain Measurements

#### Introduction
- Load carrying components for machines and structures require information about distribution of forces
- Fundamental behavior of load-carrying parts
- An object subject to load
  - Safe level of stress
  - Forces within the object balance the external loads
- A rod placed in uniaxial tension
  - A force within the material to maintain static equilibrium
  - Stress—Force per unit area
- Measuring stress directly usually not possible
  - Measuring the change in length or shape of a material

### Stress and Strain

#### Introduction

F_{F} = \frac{\sigma}{A}
\[\epsilon = \frac{\Delta L}{L}\]

#### Sub-Topics
- Lateral Strains

### Stress and Strain

#### Introduction

\[\sigma = \frac{F}{A}\]
\[\epsilon = \frac{\Delta L}{L}\]

Free-body diagram illustrating internal forces for a rod in uniaxial tension
Stress and Strain

Introduction

- For most engineering material strain is small
  - Units of $10^{-6}$ m/m or $10^{-6}$ in/in
  - Dimensionless unit of microstrain $\mu$
- Stress-strain diagrams
  - Important in understanding behavior of material under load
  - Linear relationship for loads less than that required to permanently deform
- Modulus of elasticity (Young's modulus): $E_m$
- Elastic region—Range where the relationship is linear
  - Relationship is Hook's law

Stress and Strain

Lateral Strains

- Rod stretched in axial direction
  - Cross-sectional area must decrease
  - Conservation of mass
- Rod compressed in axial direction
  - Cross-sectional area must increase
- Lateral (transverse) strain — Change in cross-sectional area
  - Ratio of change in diameter to original diameter for the rod
  - Poisson's ratio
- Components subject to loading in more than one dimension
- Relationships generalized to multi-dimensional cases

Stress and Strain

Lateral Strains

- Stress levels designed to remain well below the elastic limit for most components
- Linear relationship holds

Resistance Strain Gauges

Sub-Topics

- Metallic Gauges
- Strain Gauge Construction and Bonding
- Semiconductor Strain Gauges
Measurement of small displacements in a material or object under mechanical load
- Determines strain
- As simple as change in distance between two scribe marks
- As advanced as optical holography

The ideal sensor for strain measurement
- Good spatial resolution
- Unaffected by changes in ambient conditions
- High frequency response (dynamic measurements)
- Bounded resistance strain gauge meets these requirements
- Both metallic and semiconductor materials

Example: A common material for strain gauge is the alloy As advanced as optical holography
- Metallic and semiconductor materials
- High frequency response (dynamic measurements)
- Bounded resistance strain gauge meets these requirements
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Electrical resistivity: $\rho$  
- Example: Determine total resistance of a copper wire having diameter of 1 mm and length of 5 cm, given copper resistivity of $1.7 \times 10^{-8}$ $\Omega$ m.
  - Copper
    - $\rho_c = 1.7 \times 10^{-8}$ $\Omega$ m
    - $L = 5 \times 10^{-2}$ m
    - $D = 1 \times 10^{-3}$ m
    - $A_e = (\pi/4)D^2$
    - $A_i = (\pi/4)(0.01)^2 = 7.85 \times 10^{-7}$ m$^2$
    - The change in resistance caused by
      - Change in geometry (length and cross-section area)
      - Change in resistivity
  
  $\rho = \rho_c L/A_i$
  
  $R = (1.7 \times 10^{-8} \Omega m)(5 \times 10^{-2} m)/(7.85 \times 10^{-7} m^2)$
  
  $R = 1.3 \times 10^{-4}$ $\Omega$

Use of nickel instead
- Resistivity of $7.8 \times 10^{-6}$ $\Omega$ m
  - $\rho_n = 7.8 \times 10^{-6}$ $\Omega$ m
  - $R = \rho_n L/A_e$
  - $R = (7.8 \times 10^{-6} \Omega m)(0.00002 m)/(7.85 \times 10^{-7} m^2)$
  - $R = 4.97 \times 10^{-3}$ $\Omega$

Example: A common material for strain gauge is the alloy constantan consisting of 55% copper and 45% nickel. Typical strain gauge resistance might be 120 $\Omega$. Determine the length of constantan wire of diameter 0.025 mm, given resistivity of $49 \times 10^{-6}$ $\Omega$ m.
  - $\rho_c = 49 \times 10^{-6}$ $\Omega$ m
    - $L = 25 \times 10^{-3}$ m
    - $A_e = (\pi/4)D^2 = (\pi/4)(25 \times 10^{-3})^2 = 4.908734 \times 10^{-4}$ m$^2$
    - $R = \rho_c L/A_e$
    - $L = R A_e/\rho_c$
    - $L = (120 \Omega)(4.908734 \times 10^{-4} m^2)/(49 \times 10^{-6} \Omega m)$
    - $L = 0.12$ m
  - Wire length of 12 cm
Resistance Strain Gauges

Metallic Gauges

- A single straight conductor normally not practical
- Shown by last example
- Bend the wire conductor
  - To have several lengths of wire
  - Oriented along the axis of strain

Detail of a basic strain gauge construction

Typical metallic-foil bonded strain gauge
- Photo-etched metal foil pattern
- Mounted on plastic backing material
- Strain gauge averages measured strain over gauge length
- Designs based on applications with a variety of conditions
  - Backing material
  - Grid configuration
  - Bonding techniques (e.g., adhesives)
  - Total gauge electrical resistance
- Strain gauge backing useful for temperatures
  - Ranging from -270°C to 290°C

Construction of a typical metallic foil strain gauge

Strain gauge configurations: (a) Torque Rosette, (b) Linear Pattern, (c) Delta Rosette, (d) Residual Stress Pattern

Strain gauge configurations: (e) Diaphragm Pattern, (f) Tee Pattern, (g) Rectangular Rosette, (h) Stacked Rosette
Resistance Strain Gauges

**Strain Gauge Construction and Bonding**

- **Gauge factor**—Expresses the change in resistance of a strain gauge
  \[ GF = \frac{\delta R}{\delta x} = \frac{\delta R}{L} \]
- Empirically determined
- Supplied by manufacturer
- For metallic strain gauge \( GF \approx 2 \)

### Semiconductor Strain Gauges

- A semiconductor material changes in resistance when subjected to a load
- Used for strain measurements
- Silicon crystals: Basic material for semiconductor strain gauge
- Sliced into very thin sections
- A very large gauge factor: As large as 200
- Output nonlinear with strain
- Used for construction of very small transducers
  - Diameters less than 8 mm
  - Measurements up to 15,000 psi

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**Resistance Strain Gauges Examples**

- Strain Gauges
- Strain Gauges
- Strain Gauges
- Strain Gauges

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Strain Gauge Electrical Circuits

Introduction

- Typical gauge with sensitivity of $10^{-6}$ Ω/(kN/m²)
- High-sensitivity device such as Wheatstone bridge
  - Equipment available to measure changes less than 0.0005 Ω
  - Bridge output at initial condition: $E_0$
  - Bridge deflection: $δE$
  - Change in strain gauge resistance: $δR$
  - All fixed resistors and strain gauge resistance initially equal
  - Bridge is balanced ($E_0=0$)
  - Gauge subjected to strain

Equipment available to measure changes less than 0.0005 Ω

Bridge output at initial condition: $E_0$

Bridge deflection: $δE$

Change in strain gauge resistance: $δR$

All fixed resistors and strain gauge resistance initially equal

Bridge is balanced ($E_0=0$)

Gauge subjected to strain

Strain gauge circuit subject to uniaxial tension

Example: Strain gauge with gauge factor 2 mounted on rectangular steel bar (modulus of elasticity $200 \times 10^6$ kN/m²) that is 3 cm wide and 2 cm high, subjected to tensile force of 30 kN. Resistance with no load is 120 Ω. Determine the resistance change.

$$GF = 2$$

$$E_0 = 300 \times 10^6$$ kN/m²

$$F = 30$$ kN

$$R = 120$$ Ω

$$A = 0.03 \times 0.01$$ m²

$$\sigma = \frac{F}{A} = \frac{30}{0.03 \times 0.01} = 1 \times 10^5$$ kN/m²

$$ε = \frac{\sigma}{E} = \frac{1 \times 10^5}{200 \times 10^6} = 5 \times 10^{-4}$$ m/m

$$δR = δR = R \cdot GF = (120) \cdot (2) \cdot (5 \times 10^{-4} \cdot 2) = 0.12$$ Ω

Temperature Compensation

- Apparent strain — Any change in gauge resistance not due to the strain being measured
  - Temperature compensation
  - Eliminating certain components of strain
  - Use of identical strain gauges mounted on the top and bottom of a beam subjected to axial and bending loads
    - Gauges experience equal but opposite bending strains
    - Gauges experience the same axial strain
    - Removing the effects of bending strain

Apparent Strain and Temperature Compensation

Sub-Topics

- Temperature Compensation

Basic strain gauge Wheatstone bridge circuit
Apparent Strain and Temperature Compensation

Introduction

Strain gauge installation for bending compensation

Temperature Compensation

- Temperature compensation—Differential thermal expansion between the gauge and the material on which it is mounted
- Using gauges of identical alloy composition
- Using compensating gauges
  - Strain gauge experiencing strain and temperature strain
  - Compensating gauge experiencing only temperature strain

Apparent Strain and Temperature Compensation

Temperature Compensation

Bridge arrangements for temperature compensation

Optical Strain Measuring Techniques

Sub-Topics

- Basic Characteristics of Light
- Photoelastic Measurement
- Moire' Methods

Optical Strain Measuring Techniques

Introduction

- Optical techniques for measurement of stress and strain fields
  - Models made of material with appropriate optical properties
  - Coating techniques for existing material
- Photoelasticity—Changes in optical properties of material when subjected to strains
  - E.g., plastics
- Moire’ pattern—Optical effect resulting from transmission or reflection of light from two overlaid grid patterns
  - Fringes result from relative displacement of two grid patterns
Optical Strain Measuring Techniques

Examples

A light source emits a series of waves
- Containing vibrations in all perpendicular planes
- Effect of polarizing filter on incident light wave
  - Transmitted light is plane polarized
  - Extinction of the light beam: Second polarizing filter
  - Axis of polarization at 90 degrees to the first filter
- Behaviors of light employed to measure
  - Direction and magnitude of strain
  - In photoelastic materials

Optical Strain Measuring Techniques

Basic Characteristics of Light

A light source emits a series of waves
- Containing vibrations in all perpendicular planes
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  - Extinction of the light beam: Second polarizing filter
  - Axis of polarization at 90 degrees to the first filter
- Behaviors of light employed to measure
  - Direction and magnitude of strain
  - In photoelastic materials

Optical Strain Measuring Techniques

Photoelastic Measurement

Stress analysis accomplished
- Constructing a model of the part to be analyzed from a material selected for its optical properties
- Or by coating the actual part or prototype with a photoelastic coating
- If model constructed from suitable plastic: Required loads significantly less

Optical Strain Measuring Techniques

Photoelastic Measurement

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<table>
<thead>
<tr>
<th>Optical Strain Measuring Techniques</th>
<th>Moire’ Methods</th>
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<tbody>
<tr>
<td>Moire’ pattern—Two overlaid, relatively dense patterns that are displaced relative to each other</td>
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<tr>
<td>E.g., color printing with patterns of dots if printing slightly out of register</td>
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<tr>
<td>E.g., shimmering effect with some patterned clothing on television (size of the pattern in fabric is same as resolution of television image)</td>
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<tr>
<td>Typically a model constructed specifically for this purpose.</td>
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![Moire’ gratings](image-url)