### Strain Measurements

#### Topics

- Stress and Strain
- Resistance Strain Gauges
- Strain Gauge Electrical Circuits
- Apparent Strain and Temperature Compensation
- Optical Strain Measuring Techniques

### Strain Measurements

#### References

Strain Measurements

Introduction

- Load carrying components for machines and structures requires information about distribution of forces
- Fundamental behavior of load-carrying parts
- An object subject to load
  - Safe level of stress
  - Forces within the object balance the external loads
- A rod placed in uniaxial tension
  - A force within the material to maintain static equilibrium
- Stress—Force per unit area
- Measuring stress directly usually not possible
  - Measuring the change in length or shape of a material

Stress and Strain

Sub-Topics

- Lateral Strains
Stress and Strain
Introduction

- Experimental analysis of stress
  - Measuring deformation of a part under load
  - Inferring the existing state of stress from the measured deflections
- Cross sectional area: $A_c$
- Tension force normal to the area: $F_N$
- Normal stress
  $$\sigma_a = \frac{F_N}{A_c}$$
- Original unloaded length: $L$
- Axial strain—Ratio of change in length to the original length
  $$\varepsilon_a = \frac{\delta L}{L}$$

Free-body diagram illustrating internal forces for a rod in uniaxial tension
Stress and Strain
Introduction

- For most engineering material strain is small
  - Units of $10^{-6}$ m/m or $10^{-6}$ in/in
  - Dimensionless unit of microstrain $\mu$s
- Stress-strain diagrams
  - Important in understanding behavior of material under load
  - Linear relationship for loads less than that required to permanently deform
- Modulus of elasticity (Young’s modulus): $E_m$
- Elastic region—Range where the relationship is linear
  \[ \sigma_a = E_m \varepsilon_a \]
  - Relationship is Hook’s law

A typical stress–strain curve for mild steel
Stress and Strain
Introduction

- Stress levels designed to remain well below the elastic limit for most components
- Linear relationship holds

Stress and Strain
Lateral Strains

- Rod stretched in axial direction
  - Cross-sectional area must decrease
  - Conservation of mass
- Rod compressed in axial direction
  - Cross-sectional area must increase
- Lateral (transverse) strain—Change in cross-sectional area
  - Ratio of change in diameter to original diameter for the rod
    \[ \nu_p = \frac{|lateral\ strain|}{|axial\ strain|} = \frac{\varepsilon_L}{\varepsilon_a} \]
  - Poisson's ratio
- Components subject to loading in more than one dimension
- Relationships generalized to multi-dimensional cases
Stress and Strain
Lateral Strains

Biaxial state of stress

Resistance Strain Gauges
Sub-Topics

- Metallic Gauges
- Strain Gauge Construction and Bonding
- Semiconductor Strain Gauges
Resistance Strain Gauges

Introduction

- Measurement of small displacements in a material or object under mechanical load
  - Determines strain
  - As simple as change in distance between two scribe marks
  - As advanced as optical holography
- The ideal sensor for strain measurement
  - Good spatial resolution
  - Unaffected by changes in ambient conditions
  - High frequency response (dynamic measurements)
- Bounded resistance strain gauge meets these requirements
  - Resistance changes when deformed
  - Both metallic and semiconductor materials

Resistance Strain Gauges

Metallic Gauges

- Cross-sectional area: $A_c$
- Length: $L$
- Electrical resistivity: $\rho_e$
- Resistance of an electrical conductor
  \[ R = \frac{\rho_e L}{A_c} \]
- Subjected to normal stress along the axis of wire
  \[ \delta R = \frac{A_c (\rho_e \delta L + L \delta \rho_e) - \rho_e L \delta A_c}{A_c^2} \]
- The change in resistance caused by
  - Change in geometry (length and cross-section area)
  - Change in resistivity
Resistance Strain Gauges
Metallic Gauges

- Piezoresistance—Dependence of resistivity on mechanical strain

Example: Determine total resistance of a copper wire having diameter of 1 mm and length of 5 cm, given copper resistivity of $1.7 \times 10^{-8}$ Ωm.

Copper

\[ \rho_c = 1.7 \times 10^{-8} \, \Omega \text{m} \quad L = 5 \times 10^{-2} \, \text{m} \quad D = 1 \times 10^{-3} \, \text{m} \]

\[ A_e = (\pi/4)D^2 \]

\[ A_e = (\pi/4)(1 \times 10^{-3})^2 = 7.85 \times 10^{-7} \, \text{m}^2 \]

\[ R = \rho_c L / A_e \]

\[ R = (1.7 \times 10^{-8} \, \Omega \text{m})(5 \times 10^{-2} \, \text{m})/(7.85 \times 10^{-7} \, \text{m}^2) \]

\[ R = 1.8 \times 10^{-3} \, \Omega \]
Resistance Strain Gauges
Metallic Gauges

Use of nickel instead
Resistivity of $7.8 \times 10^{-8} \, \Omega \cdot m$
\[ \rho_c = 7.8 \times 10^{-8} \, \Omega \cdot m \]
\[ R = \rho_c L / A_c \]
\[ R = (7.8 \times 10^{-8} \, \Omega \cdot m)(5 \times 10^{-2} \, m) / (7.85 \times 10^{-7} \, m^2) \]
\[ R = 4.97 \times 10^{-3} \, \Omega \]

Resistance Strain Gauges
Metallic Gauges

Example: A common material for strain gauge is the alloy constantan (55% copper and 45% nickel). Typical strain gauge resistance might be 120 $\Omega$. Determine the length of constantan wire of diameter 0.025 mm, given resistivity of $49 \times 10^{-8} \, \Omega \cdot m$.
\[ \rho_c = 49 \times 10^{-8} \, \Omega \cdot m \quad R = 120 \, \Omega \quad D = 25 \times 10^{-3} \, m \]
\[ A_c = (\pi/4)D^2 = (\pi/4)(25 \times 10^{-3})^2 = 4.908734 \times 10^{-4} \, m^2 \]
\[ R = \rho_c L / A_c \]
\[ L = R A_c / \rho_c \]
\[ L = (120 \Omega)(4.908734 \times 10^{-4} \, m^2) / (49 \times 10^{-8} \, \Omega \cdot m) \]
\[ L = 0.12 \, m \]

Wire length of 12 cm
Resistance Strain Gauges
Metallic Gauges

- A single straight conductor normally not practical
- Shown by last example
- Bend the wire conductor
  - To have several lengths of wire
  - Oriented along the axis of strain

**Detail of a basic strain gauge construction**
Resistance Strain Gauges
Strain Gauge Construction and Bonding

- Typical metallic-foil bonded strain gauge
  - Photo-etched metal foil pattern
  - Mounted on plastic backing material
- Strain gauge averages measured strain over gauge length
- Designs based on applications with a variety of conditions
  - Backing material
  - Grid configuration
  - Bonding techniques (e.g., adhesives)
  - Total gauge electrical resistance
- Strain gauge backing useful for temperatures
  - Ranging from -270°C to 290°C

Resistence Strain Gauges
Strain Gauge Construction and Bonding

Construction of a typical metallic foil strain gauge
Strain gauge configurations: (a) Torque Rosette, (b) Linear Pattern, (c) Delta Rosette, (d) Residual Stress Pattern

Strain gauge configurations: (e) Diaphragm Pattern, (f) Tee Pattern, (g) Rectangular Rosette, (h) Stacked Rosette
### Resistance Strain Gauges
#### Strain Gauge Construction and Bonding

- **Gauge factor**—Expresses the change in resistance of a strain gauge
  
  \[ GF = \frac{\delta R}{R} = \frac{\delta L}{L} \cdot \frac{R}{\varepsilon} \]

- Empirically determined
- Supplied by manufacturer
- For metallic strain gauge GF ~ 2

### Resistance Strain Gauges
#### Semiconductor Strain Gauges

- A semiconductor material changes in resistance when subjected to a load
- Used for strain measurements
- Silicon crystals: Basic material for semiconductor strain gauge
- Sliced into very thin sections
- A very large gauge factor: As large as 200
- Output nonlinear with strain
- Used for construction of very small transducers
  - Diameters less than 8 mm
  - Measurements up to 15,000 psi
### Resistance Strain Gauges
#### Examples

![Strain Gauges](image1)

![Strain Gauges](image2)
Resistance Strain Gauges

Examples

Strain Gauges

Resistance Strain Gauges

Examples

Strain Gauges
Typical gauge with sensitivity of $10^{-6} \Omega/(kN/m^2)$

- High-sensitivity device such as Wheatstone bridge
  - Equipment available to measure changes less than 0.0005 $\Omega$
  - Bridge output at initial condition: $E_0$
  - Bridge deflection: $\delta E_0$
  - Change in strain gauge resistance: $\delta R$
    $$\frac{\delta E_0}{E_i} \approx \frac{\delta R}{R} \approx \frac{GF \epsilon}{4}$$
  - All fixed resistors and strain gauge resistance initially equal
  - Bridge is balanced ($E_0=0$)
  - Gauge subjected to strain

Basic strain gauge Wheatstone bridge circuit
Example: Strain gauge with gauge factor 2 mounted on rectangular steel bar (modulus of elasticity $200 \times 10^6$ kN/m$^2$) that is 3 cm wide and 1 cm high, subjected to tensile force of 30 kN. Resistance with no load is 120 Ω. Determine the resistance change.

\[
\begin{align*}
\sigma &= \frac{F}{A_e} = \frac{30 \text{kN}}{0.03 \text{m} \times 0.01 \text{m}} = 1 \times 10^4 \text{kN/m}^2 \\
\varepsilon &= \frac{\sigma}{E_m} = \frac{1 \times 10^4 \text{kN/m}^2}{200 \times 10^6 \text{kN/m}^2} = 5 \times 10^{-4} \text{m/m} \\
\delta R / R &= \varepsilon GF \\
\delta R &= R \varepsilon GF = (120) \times (5 \times 10^{-4}) \times (2) = 0.12 \text{Ω}
\end{align*}
\]
**Apparent Strain and Temperature Compensation**

**Sub-Topics**

- Temperature Compensation

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**Apparent Strain and Temperature Compensation**

**Introduction**

- Apparent strain—Any change in gauge resistance not due to the strain being measured
  - Temperature compensation
  - Eliminating certain components of strain
- Use of identical strain gauges mounted on the top and bottom of a beam subjected to axial and bending loads
  - Gauges experience equal but opposite bending strains
  - Gauges experience the same axial strain
  - Removing the effects of bending strain
**Apparent Strain and Temperature Compensation**

**Introduction**

Strain gauge installation for bending compensation

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**Apparent Strain and Temperature Compensation**

- Temperature compensation—Differential thermal expansion between the gauge and the material on which it is mounted
- Using gauges of identical alloy composition
- Using compensating gauges
  - Strain gauge experiencing strain and temperature strain
  - Compensating gauge experiencing only temperature strain

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Strain gauge installation for bending compensation
Apparent Strain and Temperature Compensation

Temperature Compensation

Bridge arrangements for temperature compensation

Apparent Strain and Temperature Compensation

Temperature Compensation

<table>
<thead>
<tr>
<th>Arrangement</th>
<th>Compensation Provided</th>
<th>Bridge Constant $\kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single gauge in uniaxial stress</td>
<td>None</td>
<td>$\kappa = 1$</td>
</tr>
<tr>
<td>Two gauges sensing equal and opposite strains—typical bending arrangement</td>
<td>Temperature</td>
<td>$\kappa = 2$</td>
</tr>
<tr>
<td>Two gauges in uniaxial stress</td>
<td>Bending only</td>
<td>$\kappa = 2$</td>
</tr>
<tr>
<td>Four gauges with pairs sensing equal and opposite strains</td>
<td>Temperature and bending</td>
<td>$\kappa = 4$</td>
</tr>
<tr>
<td>One axial gauge and one Poisson gauge</td>
<td>$\kappa = 1$ or $\frac{4}{3}$</td>
<td></td>
</tr>
<tr>
<td>Four gauges with pairs sensing equal and opposite strains—sensitive to tension only; typical shaft arrangement</td>
<td>Temperature and axial</td>
<td>$\kappa = 4$</td>
</tr>
</tbody>
</table>
### Optical Strain Measuring Techniques

**Sub-Topics**

- Basic Characteristics of Light
- Photoelastic Measurement
- Moire’ Methods

### Optical Strain Measuring Techniques

**Introduction**

- Optical techniques for measurement of stress and strain fields
  - Models made of material with appropriate optical properties
  - Coating techniques for existing material
- Photoelasticity—Changes in optical properties of material when subjected to strains
  - E.g., plastics
- Moire’ pattern—Optical effect resulting from transmission or reflection of light from two overlaid grid patterns
  - Fringes result from relative displacement of two grid patterns
Optical Strain Measuring Techniques
Examples

![Image of optical strain measuring technique example 1]

Optical Strain Measuring Techniques
Examples

![Image of optical strain measuring technique example 2]
Optical Strain Measuring Techniques
Basic Characteristics of Light

- A light source emits a series of waves
  - Containing vibrations in all perpendicular planes
- Effect of polarizing filter on incident light wave
  - Transmitted light is plane polarized
  - Extinction of the light beam: Second polarizing filter
  - Axis of polarization at 90 degrees to the first filter
- Behaviors of light employed to measure
  - Direction and magnitude of strain
  - In photoelastic materials

Polarization of light
Construction of a plane polariscope

Optical Strain Measuring Techniques
Photoelastic Measurement

Optical Strain Measuring Techniques
Photoelastic Measurement

Optical Strain Measuring Techniques
Photoelastic Measurement

Optical Strain Measuring Techniques
Photoelastic Measurement

Construction of a plane polariscope
Optical Strain Measuring Techniques
Photoelastic Measurement

- Stress analysis accomplished
  - Constructing a model of the part to be analyzed from a material selected for its optical properties
  - Or by coating the actual part or prototype with a photoelastic coating
- If model constructed from suitable plastic
  - Required loads significantly less than actual
  - Reduces effort and expense in testing

Optical Strain Measuring Techniques
Moire’ Methods

- Moire’ pattern—Two overlaid, relatively dense patterns that are displaced relative to each other
- E.g., color printing with patterns of dots if printing slightly out of register
- E.g., shimmering effect
  - With some patterned clothing on television
  - Size of pattern in fabric same as resolution of television image
Optical Strain Measuring Techniques
Moire' Methods

Typically a model constructed specifically for this purpose
Uses two gratings
- Patterns of parallel lines
- Spaced equally apart
Pitch—Distance between centers of lines
- From 1 to 20 lines per mm
Ratio of transparent to pitch
Greater density of lines
- Greater sensitivity
- Coherent light required
### Optical Strain Measuring Techniques
#### Moire’ Methods

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- A grating fixed directly to the surface to be studied
  - Photoengraving
- Reference grating placed in contact with the surface
- Series of fringes result
- Data reduction to determine strain
- Techniques have increased sensitivity
  - Possible grating density of 1200 lines/mm

\[
\alpha = \frac{t}{p} = 0.5 \quad \text{for this grating}
\]

\[
\alpha = \frac{t}{p} = 0.75
\]

Moire’ gratings