Math 601 Homework #2. Due Wed., Jan. 31, 2018

Read Chapter 2 of the textbook (Bierbrauer)

Most of the exercises in the text are quick and easy, and you should try them all, to make sure you get the basics. I will assign some of the less routine exercises.

Write up and hand in solutions to the following exercises. You must give complete justification for all answers.

Exercises in textbook, Chapter 1

Sec. 1.6, p. 21, #1.6.2, 1.6.3, 1.6.9

Additional exercises (to hand in)

1. Find all the error patterns that the code $C = \{101, 011, 111\}$ corrects.
2. Let $C = \{000000, 111110, 101101, 010111\}$.
   (a) Does C correct $e = 110000$?
   (b) Does C correct $e = 101000$?
3. Let $C = \{000000, 101010, 010101, 111111\}$.
   (a) What is $d(C)$?
   (b) For what value of u is C exactly-u-error-correcting?
   (c) Find an error pattern of weight $\lfloor (d-1)/2 \rfloor + 1$ that C corrects.
   (d) Find an error pattern of weight $\lfloor (d-1)/2 \rfloor + 1$ that C does not correct.
4. Prove: Let $C \subseteq \mathbf{F}^n$, $e \in \mathbf{F}^n$, and $e \neq \mathbf{0}$. If C corrects e, then C detects e.
5. Find an upper bound for the number of codewords of a code with
   (a) $n = 8$, $d = 3$
   (b) $n = 15$, $d = 5$
6. Show that there is no perfect code with $n = 10$ and $d = 5$.

Extra Credit:

1. p. 21, #1.6.8
2. Prove: Let $C \in \mathbf{F}^n$ be a code and e and f be error patterns in $\mathbf{F}^n$. Suppose, for all i, $e_i = 1$ implies $f_i = 1$. If C corrects f, then C corrects e.