Math 601 Homework #3. Due Wed., Feb. 7, 2018

Read Chapter 2 of the textbook (Bierbrauer)

Most of the exercises in the text are quick and easy, and you should try them all, to make sure you get the basics. I will assign some of the less routine exercises.

Write up and hand in solutions to the following exercises. You must give complete justification for all answers.

Exercises in textbook, Chapter 2

Sec. 2.1, p. 26, #2.1.1, 2.1.3

Additional exercises (to hand in)

1. For each of the following codes, decide if it is a linear code. Justify your answers.
   (a) $C = \{010, 001, 011, 111\}$
   (b) $C = \{000, 110, 001, 111\}$
   (c) $C = \{0000, 1100, 1010, 1001, 0110, 0101, 0011\}$
   (d) $C = \{00000, 11111\}$
   (e) $C = \{00000, 11011, 11100, 00111\}$
   (f) $C = \{0000, 1100, 1011, 0110\}$
2. For each of the linear codes in exercise 1, give the minimum distance $d(C)$.
3. For each of the following sets S, list the elements of the linear code $\mathrm{span}(S)$.
   (a) $S = \{010, 011, 111\}$
   (b) $S = \{1010, 0101, 1111\}$
   (c) $S = \{11000, 01001, 01111, 10111\}$
4. For each of the sets S in exercise 3, determine if S is linearly dependent or linearly independent. If S is linearly dependent, write a linear dependence of its elements.
5. For each of the linear codes $\mathrm{span}(S)$ in exercise 3 give a basis for the code.
6. Write each of the following words in $\mathbf{F}^4$ as a unique linear combination of the words in the basis $\{1000, 1100, 1110, 1111\}$.
   (a) 0011
   (b) 1010
   (c) 0000
   (d) 1001
7. Find a basis for $\mathbf{F}^4$ that contains 1010 and 1110.

8. True or False? If true, explain why. If not, give a counterexample.

   (a) If C and D are binary linear codes of the same length, then $C \cap D$ is a binary linear code.

   (b) If C and D are binary linear codes of the same length, then $C \cup D$ is a binary linear code.

   (c) If S is a set of three strings in $\mathbf{F}^n$, then $\text{span}(S)$ is a linear code of dimension 3.

Extra Credit:

1. Let C_1 be a binary linear code of length m and dimension j and C_2 a binary linear code of length n and dimension k. Consider the length $m+n$ code

   $C = \{uv : u \in C_1 \text{ and } v \in C_2\}$. Prove that C is a linear code. What is the dimension of C? Prove your answer.

2. Prove: If C is a binary linear code, then either all words of C have even weight, or exactly half the words of C have even weight.