Math 601 Homework #7. Due Wed., Mar. 14, 2018

Write up and hand in solutions to the following exercises. You must give complete justification for all answers.

1. Let C be the extended Hamming code of length 8, that is,

   $C = \{vj : v \in \mathcal{H}_3 \text{ and } v_1 + v_2 + \cdots v_7 + j = 0\}$.

   (a) Give a parity-check matrix for C.

   (b) Decode the received words 10001010, 11010110, 11111111.

   (c) Show that C is self-dual.

2. Show that the length 24 vector of all ones is in the extended Golay code, C_{24}.

3. Use #2 to show that C_{24} contains no words of weight 20.

4. Decode the word $w =$ 011 110 110 100, 010 000 010 010 in C_{24}. Refer to Extended Golay Code and the Golay Code for the parity-check matrices (and matrix B) to use.

5. In the following we give the syndromes of C_{24}, s and s', computed for a received word. Apply the decoding algorithm for the extended Golay code to determine the error. Refer to Extended Golay Code and the Golay Code for the matrix B to use.

   (a) $s =$ 010 010 100 101, $s' =$ 001 000 110 000

   (b) $s =$ 101 010 110 101, $s' =$ 001 111 110 100

6. Decode the word $w =$ 010 100 010 000, 010 010 000 00 in C_{23}. Refer to Extended Golay Code and the Golay Code for the parity-check matrices (and matrix B) to use.

Extra credit:

1. A *Steiner triple system*, $S(t, k, v)$ is a collection of k-element subsets of a v-element set A such that every t-element subset of A is in exactly one of the k-elements subsets. Explain, with justification, how C_{24} gives a $S(5, 8, 24)$ Steiner triple system. (Hint: Consider words in $\mathbf{F}^{24}$ of weight 5.)

2. Use the fact that every word of weight 4 in $\mathbf{F}^{23}$ is distance exactly 3 from a codeword to count the number of words of weight 7 in the Golay code C_{23}. (Hint: for any codeword c of weight 7, the number of words that have weight 4 and are distance 3 from c is $\binom{7}{3}$.)