Math 601 Homework Assignment #9 Due Wednesday, April 4, 2018

Write up and hand in solutions to the following exercises. You must give complete justification for all answers. Assume all polynomials are in $\mathbf{F}[x]$. Refer to the Factorization list in Blackboard or at http://people.ku.edu/ bayer/601factorization.pdf

1. Prove: If C is a linear cyclic code, and C detects the error pattern e, then C detects $\pi(e)$.

2. For each of the words below, find the generator polynomial for the smallest linear cyclic code containing that word.

   (a) 0101100

   (b) 010101

3. Let C be the linear code with generator matrix

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

   (a) Show C is cyclic.

   (b) Find the generator polynomial for C.

4. Find generator polynomials for three linear cyclic codes of length 15, one each of dimension 5, 6, and 9.

5. Find the dimensions of all linear cyclic codes of length

   (a) 17 (b) 10

6. Let C be the linear cyclic code of length 7 with generator polynomial $g(x) = 1 + x + x^2 + x^4$.

   (a) Give the check polynomial for C.

   (b) Use the check polynomial to determine if the following represent codewords in C: $w_1(x) = 1 + x^2 + x^5 + x^6$ and $w_2(x) = 1 + x^4 + x^5$.

7. For each of the (n, k) below, answer the following, justifying your answers.

   (a) Is there a linear cyclic code of length n and dimension k?

   (b) If there is such a code, give the generator polynomial and check polynomial for one such code C.

   (i) $(n, k) = (15, 9)$ (ii) $(n, k) = (13, 8)$ (iii) $(n, k) = (14, 9)$

8. Let C be the linear cyclic code of length 7 with generator polynomial $g(x) = 1 + x^2 + x^3$.

   (a) Give the transposed parity-check matrix H^T having $x^j \bmod g(x)$ in its jth row.

   (b) Say why C is a Hamming code.

9. For each of the following n and $g(x)$, find the generator polynomial for the dual code to the code of length n and generator polynomial $g(x)$.

   (a) $n = 7$, $g(x) = 1 + x + x^2 + x^4$

   (b) $n = 9$, $g(x) = 1 + x^3 + x^6$

10. Prove: Let C be a linear cyclic code. If C contains a codeword of odd weight, then the generator polynomial of C corresponds to a codeword of odd weight.

Extra credit:

1. Prove: If C is a linear cyclic code, and C corrects the error pattern e, then C corrects $\pi(e)$.

2. Prove: Let C be a linear cyclic code of length $n \geq 3$ with generator polynomial $g(x)$ of degree at least 1. If n is the least positive integer such that $g(x)$ divides $1 + x^n$, then C has minimum distance at least 3.

3. Prove: For each $n \geq 3$, n not a power of 2, there is a polynomial (in $\mathbf{F}[x]$) $f(x) = f_0 + f_1 x + \cdots f_k x^k$ of degree $k \geq 1$ such that $f(x)$ divides $1 + x^n$ and $(f_0 f_1 \ldots f_k)$ has odd weight.