Math 601 Homework Assignment #10 Due Wednesday, April 11, 2018

Write up and hand in solutions to the following exercises. You must give complete justification for all answers. Assume all polynomials are in $\mathbf{F}[x]$. Refer to the Factorization list in Blackboard or at http://people.ku.edu/ bayer/601factorization.pdf

1. (a) How many polynomials in $\mathbf{F}[x]$ have degree (exactly) 3?
   (b) List all polynomials of degree (exactly) 3 in $\mathbf{F}[x]$.
   (c) For each polynomial $f(x)$ of degree 3, factor it into irreducible factors or identify $f(x)$ as irreducible itself.
2. Let $g(x) = 1 + x^2 + x^3$ and $K = \mathbf{F}[x]/(g(x))$. Give the multiplication table for $K \setminus \{0\}$.
3. Determine whether each of the following polynomials is irreducible; if it is, determine whether it is primitive. If it is not primitive, give $m < 2^r - 1$ such that the polynomial divides $1 + x^m$.
   (a) $1 + x + x^4$
   (b) $1 + x + x^2 + x^4$
   (c) $1 + x + x^5$
   (d) $1 + x^2 + x^5$
   (e) $1 + x^3 + x^6$

   (Hint: Use the table of factorizations of $1 + x^n$ for the primitive question.)

Extra credit:

1. Prove: Let $g(x)$ be a primitive polynomial of degree r, and let $\beta = x \bmod g(x)$. Then β^i is primitive if and only if β^{2^r-1-i} is primitive.
2. Prove: Let $g(x)$ be a primitive polynomial of degree r, and let $\beta = x \bmod g(x)$. Then β^i is primitive if and only if $\gcd(i, 2^r - 1) = 1$.