Math 601 Homework Assignment #13 Due Wednesday, May 2, 2018

Write up and hand in solutions to the following exercises. You must give complete justification for all answers.

1. Write generator matrices for the Reed-Muller codes, $RM(2,3)$ and $RM(1,4)$.
2. The Reed-Muller code $RM(1,3)$ has generator matrix $G(1,3)$ below.

$$G(1,3) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

   (a) Write a parity-check matrix for $RM(1,3)$.

   (b) What is the distance, $d(RM(1,3))$?

   (c) Give the syndrome table for IMLD for $RM(1,3)$ using your parity-check matrix. (That is, include errors of weight 0 and 1 only.)

   (d) Decode $w_1 = 01100111$.

   (e) Show that $w_2 = 00010100$ is not within distance 1 of any codeword, and find two codewords c such that $d(w_2, c) = 2$.

3. For $c \in \mathbf{F}^n$, let $\bar{c}$ be the word obtained by replacing every 0 in c by a 1 and every 1 in c by a 0.

   Prove: If $c \in RM(r,m)$, then $\bar{c} \in RM(r,m)$.

4. Prove: $RM(r,m)$ is self-dual if and only if $m = 2r + 1$.
5. (Generalization of the construction of the Reed-Muller codes)

   Let C_1 be a $[n, k_1, d_1]_2$ code and C_2 be a $[n, k_2, d_2]_2$ code.

   Define $C = \{(x, x + y) : x \in C_1, y \in C_2\}$. Show that C is a $[2n, k_1 + k_2, d]_2$ code, where $d = \min\{2d_1, d_2\}$. (Hint: for the distance, consider codewords $(x, x + y)$ of three types: $x = y \neq 0$; $x \neq y$ and $y \neq 0$; and $y = 0$ and $x \neq 0$.)

Extra Credit on next page.

Extra credit:

The ternary Golay code can be obtained from a length 12 code by puncturing, in the same way as we got the binary Golay code from the extended Golay code. Let C be the code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 2 & 1 & 2 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 2 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 2 & 2 & 1 \end{bmatrix}.$$

Note: This is corrected from an earlier version!

1. Show that the ternary Golay code is obtained from C by deleting the last symbol from each codeword.
2. Show that C is a $[12, 6, 6]_3$ code. (You may assume the ternary Golay code is distance 5.)
3. Show that C is self-dual.