Soc 760: Social Inequality
Inequality Indexes

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Types of Inequality Effects

1. Mechanical consequences
2. Relational Effects
3. Functional Form Effects
4. Externality Effects

Mechanical Consequences

$$y_i = a + \beta x_i + e_i \quad (1)$$

where y_i is an outcome and x_i is an income for individual i.

$$\sigma_y^2 = \beta^2 \sigma_x^2 + \sigma_e^2 \quad (2)$$

If the variance of x increases, the variance of y increases in proportion to the square of β. E.g., x_i is log earnings of fathers and y_i is log earnings of sons.

Relational Effects

$$\sigma_y^2 = \beta^2 \sigma_x^2 + \sigma_e^2 \tag{3}$$

When β is increasing, even though σ_x^2 is fixed, σ_y^2 grows. E.g., the effect of income on subjective well-being rose between 1975 and 2000 (Hout 2003).

Functional Form Effects

y is related to income in a nonlinear form.
E.g., Atkinson index.

Externality Effects

No direct consequence of x on y, but through other indirect effects.

1. Psychological effects: Relative deprivation; Relative gratification
2. State spending: median voter theory
3. Social organization: economic inequality $\rightarrow$ economic segregation $\rightarrow$ child health, education etc.

Lorenz Curve

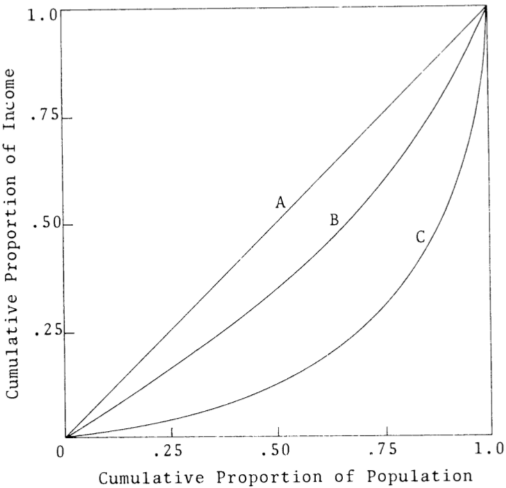

Figure 1. Lorenz Curves for Three Distributions of Income

Condition of Inequality Index

1. It should be zero when all individuals have identical incomes, and should have a positive value when two or more individuals differ.
2. Scale invariance: multiplying everyones income by a constant leaves the degree of inequality unchanged.
    1. No change with weight (ex. dollar = yen)
    2. Proportionate increases should leave inequality unchanged. (No change in relative differences.)
    3. Constant addition (or subtraction) should be counted.
3. Principle of transfer: measures of inequality ought to increase whenever income is transferred from a poorer person to a richer person; D, L do not match this principle.

Coefficient of Variation (COV)

$$V = \frac{\sigma}{\mu} \tag{4}$$

- Standard deviation divided by the mean.
- Min $= 0$ and Max $= \infty$.
- $V/(V+1)$ to make the COV have an upper bound of 1.

Relative Mean Deviation (RMD)

$$D = \frac{\frac{1}{n}\sum_{i=1}^{n}|x_i - \mu|}{2\mu} \tag{5}$$

D is not affected by transfer between both above the mean or both below the mean

Gini Index

$$
\begin{aligned}
G &= \frac{\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}|x_i - x_j|}{2\mu} \\
&= \frac{2}{\mu n^2}\sum_{i=1}^{n} i x_i - \frac{n+1}{n}
\end{aligned}
\tag{6}
$$

- Gini index is a measure of dispersion divided by twice the mean. Gini coefficient is the average absolute difference between all pairs of individuals.
- Min = 0 and Max = 1.
- To use *Gini* as a dependent variable, transform it to logit, $\ln \frac{G}{1-G}$.

Theil Index

$$T = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\mu}\right)\ln\left(\frac{x_i}{\mu}\right) = \frac{\left[\frac{1}{n}\sum_{i=1}^{n}(x_i \ln x_i)\right] - \mu \ln \mu}{\mu} \tag{7}$$

A measure of dispersion divided by the mean.
Min $= 0$ and Max $= \infty$.

Variance of Log Earnings (VarLog)

$$L(VarLog) = \frac{1}{n}\sum_{i=1}^{n}(\ln x_i - \bar{\ln x}) \\ = \frac{1}{n}\sum_{i=1}^{n}(z_i - \bar{z}) \tag{8}$$

where z_i is log-transformed earnings.

Easy to compute.

L is undefined when the distribution includes incomes of zero.

At high income levels (greater than 2.718 times the geometric mean), L decrease with a transfer from a poorer to a richer.

Geometric Mean

Arithmetic Mean $= \frac{1}{n}\sum_{i=1}^{n} x_i$
Geometric Mean $= \sqrt[n]{x_1 x_2 \cdots x_n}$
$\ln GeoMean = \frac{1}{n}\sum_{i=1}^{n} \ln x_i$. Therefore, the log of the geometric

mean is the arithmetic mean of the logs of the numbers.

Sensitivity to Transfers

1. *COV*: equally sensitive to transfers at all income levels.
2. *Gini*: most sensitive to transfers around the middle of the distribution and least sensitive to transfers among the very rich or the very poor.
3. *Theil*: the change in T depends on the ratio of the incomes (x1/x2, if h income transfer from x1 to x2). The lower the level of income, the more sensitive T is to transfer. That is, more sensitive to the income transfer from the extremely poor to the extremely rich.
4. Which index is better depends on the context. *COV* is preferable when measures of inequality of age, city size, years of schooling (variable has no diminishing marginal effect.) *Theil* is appropriate when there is diminishing marginal effect.

Decomposition

Decomposition of inequality into between and within group inequalities.

$$Theil = \sum_j s_j T_j + \sum_j s_j \ln \frac{\bar{x}_j}{\bar{x}} \quad (9)$$

where s_j refers to the share of group j (i.e., %).

The first component refers to the within-group inequality and the 2nd component refers to the between-group inequality.
(See Kim and Sakamoto 2008 *ASR* for detail.)

Summary

1. *Gini* index: most popular but most difficult to compute and cannot be decomposed.
2. *Theil*: desirable for measuring inequality of income, or other social rewards having diminishing marginal utility.
3. *COV*: good for no diminishing marginal utility such as age.

None of these inequality measures is appropriate for interval-level variables which lack a theoretically fixed zero.

Social Welfare and Inequality Index

$$W = \sum_{i=1}^{n} U(x_i) \tag{10}$$

where $U(x_i)$ refers to the utility of income for individual i.

Two Principles and consequence

1. All individuals have the same utility function (i.e., $U(x_1) = U(x_2) = \cdots = U(x_n)$).
2. Diminishing marginal utility from increasing income (i.e., $U(x_i)$ is concave).
3. Under these conditions, W will be maximized when all incomes are equal.

Social Welfare and Inequality Index

1. Under these conditions, an ordering of the Lorenz curves = an ordering of social welfare.
2. Given a fixed total income, if the Loren curve for one distribution X lies above the Lorenz curve for another distribution Y, X is preferable to Y under a broad class of functions defining preferability.

Atkinson Index (or Social Welfare Index)

$$A = 1 - \frac{1}{\mu}\left(\frac{1}{n}\sum_{i=1}^{n} x_i^{1-e}\right)^{\frac{1}{1-e}} \text{ when } e > 0 \text{ and } e \neq 1 \qquad (11)$$

$$A = 1 - \frac{1}{\mu}\left(\prod_{i=1}^{n} x_i\right)^{1/n} \text{ when } e = 1 \qquad (12)$$

1. e refers to inequality aversiveness. As e rises, A becomes more sensitive to transfers among lower incomes and less sensitive to transfers among top income recipients.
2. As $e \to 1$, A goes to 1 - M/μ where M is the geometric mean and μ is the arithmetic mean.
3. Atkinson index is scale invariant and satisfies the principle of transfers.
4. Min $= 0$, Max $= 1$.