# Week 8. Logistic Regression 2

This handout is heavily indebted on the following two references:

- Powers, Daniel A. and Yu Xie. 2000. Statistical Methods for Categorical Data Analysis. Academic Press.
- Mize, Trenton D. 2019. "Best Practices for Estimating, Interpreting, and Presenting Nonlinear Interaction Effects." *Sociological Science* 6: 81-117.

Install spost13\_ado.pkg in your machine. Type net install spost13\_ado.pkg in Stata. spost13\_ado.pkg is a user-written program developed by Scott Long at Indiana University.

## Logit

- 1. Recall that logit = log odds = log  $\frac{P_A}{1-P_A}$
- 2. logit =  $b_0 + b_1 x_1 + b_2 x_2$ ,
- 3.  $\exp(\log \text{ odds}) = \text{ odds} = e^{b_0 + b_1 x_1 + b_2 x_2}$
- 4. odds =  $e^{b_0}e^{b_1x_1}e^{b_2x_2}$

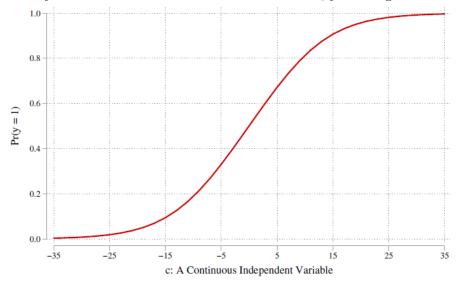
are added.

- 5. Thus, as  $x_1$  increases by 1 unit, odds of event A increases by  $\exp(b_1)$  times, net of  $x_2$ .
- 6.  $\exp(b_1)$  is called odds ratio.
- 7. Compared to the reference group (or as  $x_1$  increases by 1 unit), the likelihood of event A is  $\exp(b_1)$  times more likely. (Note that here likelihood implies odds.)
- 8. If you would like to discuss the effect of x on  $P_A$  rather than the effect of x on the odds of A, you should report marginal effects.

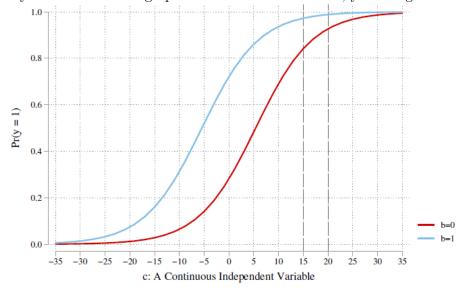
#### Logit: Interaction effects without interaction terms

- 1. The effects of independent variables are linear on logit ( $= \log \text{ odds}$ ).
- 2. Because all logit functions are multiplicative, the effect of independent variable on P are nonlinear even though there is no interaction effects. That is, the effect of x on P varies across x.
- 3. Suppose you have the following result:  $\log \text{ odds} = -1 + 0.2 \text{ c} + 2 \text{ b}$  where "c" is a continuous variable and "b" is a binary dummy variable. No interaction terms

4. When you draw the cdf of the continuous variable, you will get the following graph:



- 5. As you see, the effect of "c" on P varies across "c". At c=-25, the slope of c (or 1st derivative or the effect of c) is small (or very flat), but at c=0, the slope of c is quite steep.
- 6. If you draw the same graph for both b=0 and b=1, you will get the following graph:



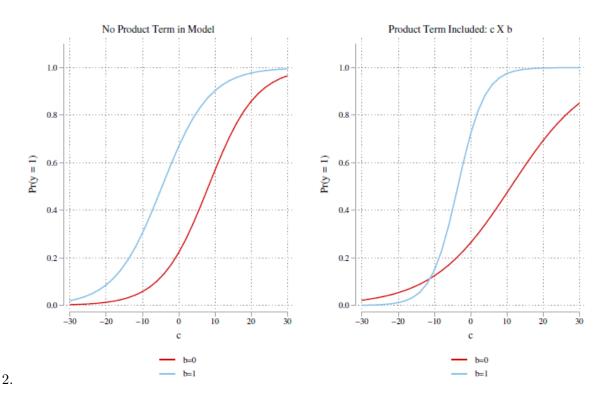
- 7. The effect of "b" varies across "c". In the middle of the distribution c, the effect of b is substantial, while the effect of b is tiny at the both ends of the distribution c.
- 8. The ceiling and flooring effects of logit (and probit) is called "compression." In essence, the effect of independent variables on P is always interactive even without interaction terms.
- 9. The effect of "c" differs between b = 0 and b = 1 depending on "c". In the middle, the slopes of c are basically the same between d = 0 and d = 1. At the high end, the slope of c is steep for b = 0 while it is flat for b = 1.
- 10. Solution 1: Report marginal effects using the margins command which is an average effect.
- 11. Solution 2: Draw a graph.

12. Solution 3: Compute the marginal effects at different points of interest, using the margins and lincom (and mlincom) command.

```
margins if b==0, at(c=5)
margins if b==1, at(c=5)
mlincome 2-1
```

## Logit: Interaction effects with interaction terms

1. Now we add an interaction term on the logit model and get the following coefficients:  $\log \text{ odds} = -1 + 0.2c + 2b + 0.2(c \times b)$ 



- 3. The significance of the coefficient estimated for the interaction term does not necessarily indicate whether the interaction is statistically significant. The statistical significance of the interaction term varies across independent variables.
- 4. Solution 1: The best way to do is to present a graph.
- 5. Solution 2: Compute the marginal effects at different points of interest, using the margins and lincom (and mlincom) command.

### Example:

- Dependent variable: Having a managerial or professional occupation.
- Independent variable: female, education(BA+), racial minority, and age

```
. logit profmanag minority female ba age age2 [pw=perwt]
Logistic regression
                                                Number of obs
                                                                      22256
                                                Wald chi2(5)
                                                                     1980.90
                                                Prob > chi2
                                                                      0.0000
Log pseudolikelihood = -429223.45
                                                Pseudo R2
                                                                      0.1288
                            Robust
                   Coef. Std. Err.
                                      z
                                                        [95% Conf. Interval]
   profmanag |
                                              P>|z|
   minority | -.5967156
                           .0423213
                                    -14.10
                                              0.000
                                                       -.6796638
                                                                   -.5137674
     female |
               .1127462
                          .0378161
                                      2.98
                                              0.003
                                                        .038628
                                                                   .1868644
                2.430903
                          .0561393
                                      43.30
                                              0.000
                                                                   2.540934
         ba |
                                                       2.320872
        age | -.1344215
                           .2073944
                                      -0.65
                                              0.517
                                                       -.540907
                                                                    .272064
                                       0.58
                                                       -.0036149
        age2 |
                .0015181
                          .0026189
                                              0.562
                                                                    .006651
                                       0.51
       _cons |
                2.093365
                           4.089812
                                              0.609
                                                       -5.922518
                                                                   10.10925
```

Non-significant interaction effect: minority \* female

. logit profmanag	i.minority##	#i.female ba	age age	2 [pw=per	wt]	
Logistic regression	on			Number o	f obs =	22256
				Wald chi	2(6) =	1985.82
				Prob > c	hi2 =	0.0000
Log pseudolikeliho	ood = -429194	1.32		Pseudo R	.2 =	0.1288
		Robust				
   profmanag 		Std. Err.			[95% Con:	f. Interval]
		Std. Err.				
1.minority		Std. Err.  .0617447	-8.93	0.000	6724239	4303892
1.minority	5514066	Std. Err.  .0617447	-8.93	0.000	6724239	4303892
1.minority   1.female           	5514066	Std. Err. .0617447 .0444581	-8.93 3.12	0.000 0.002	6724239	4303892 .2259219
1.minority   1.female   	5514066 .1387856	Std. Err0617447 .0444581 .0844265	-8.93 3.12 -1.05	0.000 0.002 0.296	6724239 .0516493 2537102	4303892 .2259219 .0772354

```
age2 | .0015159 .0026192 0.58 0.563 -.0036176 .0066495

_cons | 2.082047 4.090049 0.51 0.611 -5.934301 10.0984
```

Significant interaction effect: minority \* ba

```
. logit profmanag i.minority##i.ba female age age2 [pw=perwt]
                                       Number of obs =
                                                         22256
Logistic regression
                                       Wald chi2(6) = 2072.96
                                       Prob > chi2 =
                                                       0.0000
Log pseudolikelihood = -428973.94
                                       Pseudo R2
                                                         0.1293
______
                      Robust
 profmanag | Coef. Std. Err. z P>|z| [95% Conf. Interval]
 1.minority | -.6460647 .046621 -13.86 0.000 -.7374402 -.5546892
      1.ba | 2.300923 .0665198 34.59 0.000 2.170547
                                                       2.4313
minority#ba |
      1 1 | .3935145 .119472 3.29 0.001 .1593537 .6276754
    \texttt{female} \ | \quad .112484 \quad .0378229 \qquad 2.97 \quad 0.003 \quad .0383524 \quad .1866156
      age | -.1390151 .207292 -0.67 0.502
                                             -.5453 .2672698
      age2 | .0015759 .0026176 0.60 0.547
                                             -.0035545 .0067062
                     4.08792 0.54 0.591
                                           -5.81388 10.21047
            2.198296
     _cons |
```

Computing marginal effects (or predicted probabilities)

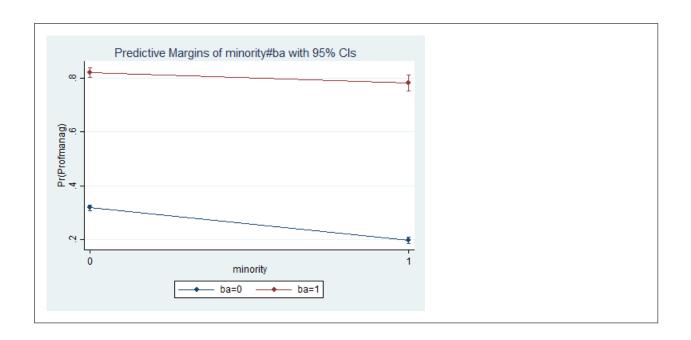
inority#ba	!						
0 0	ı	.3176858	.0050686	62.68	0.000	.3077515	.3276202
0 1		.822589	.0090821	90.57	0.000	.8047885	.8403895
1 0	1	.1962522	.0063493	30.91	0.000	.1838079	.2086966
1 1	1	.7827279	.0154165	50.77	0.000	.7525121	.8129438

Testing the difference between majority BA (2nd line) and minority BA (4th line)

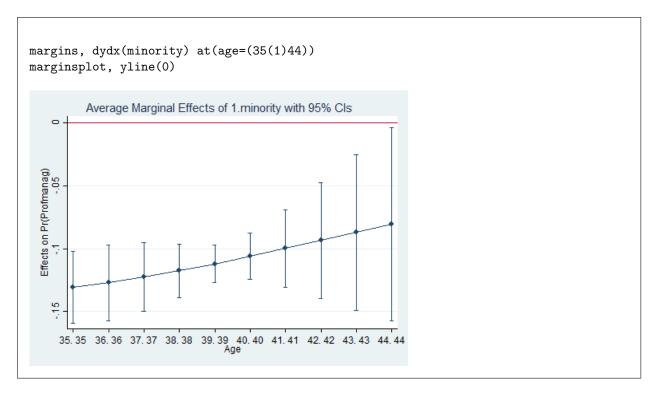
Testing the difference between (A) majority BA (2nd line) and minority BA (4th line) and (B) majority non-BA (1st line) and minority non-BA (3rd line).

Drawing a graph about the interaction effect between minority and BA+

```
margins minority#ba
marginsplot
```



Drawing a graph on the marginal effect of being minority over age



#### **Multinomial Logit**

- 1. Multinomial Logit models (mlogit) can be used when your outcome is multinomial (= 3 or more discrete choices).
- 2. For logit models, the probability distribution of the event A given the total n follows the binomial distribution. For multinomial logit, the probability distribution of the count  $Y_j$  given the total n follows the multinomial distribution.
- 3. Suppose there are 3 possible outcomes:  $Y_1$ ,  $Y_2$ , and  $Y_3$ . (more generally, we can say that there are J possible outcomes  $Y_1$ ,  $Y_2$ , ... and  $Y_J$ ).
- 4. Then, we can estimate the log odds of event j compared to the reference point J as follows:

$$\log \frac{P_j}{P_I} = \alpha_j + x'\beta_j$$

Note that  $\frac{P_j}{P_J}$  is the odds of  $Y_1$  compared to  $Y_3$ . In logit,  $\frac{P_A}{1-P_A}$  is the odds of event A compared to event not-A. Thus, logit is a special case of multinomial logit.

If we set  $Y_3$  as a reference event, we can estimate the following two models:

$$\log \frac{P_1}{P_3} = \alpha_1 + x \prime \beta_1$$

$$\log \frac{P_2}{P_3} = \alpha_2 + x / \beta_2$$

Each coefficient estimated quantifies the change in log odds of  $Y_j$  compared to  $Y_J$  when x increases by 1 unit. Put differently,  $\exp(\beta)$  indicates the odds ratio of  $Y_j$  compared to  $Y_J$  when x increases by 1 unit. As x increases by 1 unit, the likelihood of  $Y_j$  compared to  $Y_J$  increases by  $\exp(\beta)$  times.

5. The estimation of  $\log \frac{P_1}{P_2}$  is not necessary because all coefficients for  $\log \frac{P_1}{P_2}$  can be computed from the previous 2 models.

$$\log \frac{P_1}{P_3} - \log \frac{P_2}{P_3} = (\log P_1 - \log P_3) - (\log P_2 - \log P_3) = \log P_1 - \log P_2 = \log \frac{P_1}{P_2}$$

thus,

$$\log \frac{P_1}{P_2} = (\alpha_1 - \alpha_2) + x\prime(\beta_1 - \beta_2)$$

6. Estimation of the parameters of this model by maximum likelihood proceeds by maximization of the multinomial likelihood with the probabilities  $P_j$  viewed as functions of the  $\alpha_j$  and  $\beta_j$  parameters.

# Multinomial Logit Example

 $\bullet$  Dependent Variable: Employment Status = (1) Employed, (2) Unemployed, (3) Not in Labor Force.

			3.7	1 .		0056
Multinomial logistic	regression			mber of o ld chi2(1		2256 1.83
				ob > chi2		0000
Log pseudolikelihood	l = -403878 3	8		eudo R2		0978
log pseudolikelinood	400070.00	3	150	sudo Itz	0.	0310
 I		Robust				
empstat	Coef.	Std. Err.	z 	P> z  	[95% Conf.	Interval]
1Employed	(base outco	ome)				
2Unemployed $ $						
edu						
2	2816744	.1293254	-2.18	0.029	5351475	0282013
3		.1303798	-3.90	0.000	7643069	2532274
4	-1.216158	.143058	-8.50	0.000	-1.496547	9357697
5   I	-1.692009	.1765734	-9.58	0.000	-2.038086	-1.345931
female	.2232413	.0794099	2.81	0.005	.0676006	.3788819
minority $ $		.0842396	5.43	0.000	.2925873	.6228005
age		.4206603	-1.45	0.148	-1.433179	.2157791
age2		.0053096	1.44	0.150	0027648	.0180486
_cons	9.629997	8.300133 	1.16	0.246	-6.637965 	25.89796
3Not_in_labor_~e						
edu						
2		.0944152	-6.75	0.000	8227608	45266
3	2.020020	.0909344	-11.44	0.000	-1.218743	8622865
4	-1.089392	.0942667	-11.56	0.000	-1.274151	9046326
5   	-1.718559	.1094104	-15.71	0.000	-1.932999	-1.504119
female		.0643168	31.28	0.000	1.885878	2.137995
minority $ $		.0568998	-0.78	0.433	1561653	.0668779
age		.2787946	0.49	0.626	4104703	.6823844
age2	0017603	.0035227	-0.50	0.617	0086647	.0051441
_cons	-4.806532	5.497639	-0.87	0.382	-15.58171	5.968642

Computing the probability of employment by education.

```
. margins edu, atmeans
Adjusted predictions
                                                 Number of obs =
                                                                         22,256
Model VCE : Robust
1._predict : Pr(empstat==1__Employed), predict(pr outcome(1))
2._predict : Pr(empstat==2__Unemployed), predict(pr outcome(2))
3._predict : Pr(empstat==3__Not_in_labor_force), predict(pr outcome(3))
                       = .0918028 (mean)
at
             : 1.edu
                              = .2230475 (mean)
               2.edu
                                  .2969211 (mean)
               3.edu
                              =
               4.edu = .2437092 (mean)
5.edu = .1445194 (mean)
0.female = .4966023 (mean)
1.female = .5033977 (mean)
               0.minority = .6600245 (mean)
1.minority = .3399755 (mean)
                               = 39.54809 (mean)
               age
                               = 1572.167 (mean)
               age2
                         Delta-method
                 Margin Std. Err. z P>|z| [95% Conf. Interval]
_predict#edu |
        1 1 | .7059303 .0142476 49.55 0.000
                                                          .6780054 .7338551
        1 2 | .8024339 .0075069 106.89 0.000
                                                          .7877206 .8171472

    .850526
    .0055411
    153.49
    0.000
    .8396656
    .8613864

    .8806488
    .0050144
    175.62
    0.000
    .8708207
    .8904769

    .9300099
    .0045712
    203.45
    0.000
    .9210505
    .9389694

        13 |
        1 4 l
        1 5 | .9300099
        2 1 | .0813647
                            .0080513 10.11 0.000
                                                          .0655845 .0971449
        2 2 |
                 .0697837
                            .0047661
                                        14.64 0.000
                                                           .0604423
                                                                        .079125
        23 |
               .0589396
                                      15.21 0.000
                            .0038759
                                                          .0513429
                                                                      .0665362
        2 4 | .030082
                            .0026606 11.31 0.000
                                                          .0248672 .0352967
        2 5 | .0197393
                            .0027065 7.29 0.000
                                                          .0144347 .0250439
                            .0129952 16.37 0.000
        3 1 |
                 .212705
                                                            . 187235
                                                                       .2381751
                                      20.43 0.000
        3 2 | .1277824
                            .0062557
                                                        .1155215
                                                                     .1400433
        3 3 | .0905344 .0042333 21.39 0.000
                                                          .0822373 .0988316
        3 4 | .0892692 .0043666 20.44 0.000
                                                          .0807109 .0978275
        3 5 |
                 .0502508 .0037632 13.35 0.000
                                                            .042875
                                                                        .0576266
```

Gender comparison across the probabilities of three outcomes.

```
. margins edu, predict() over(i.female) atmeans
```

```
Adjusted predictions
                                             Number of obs
                                                                   22,256
Model VCE
           : Robust
over
            : female
            : Pr(empstat==1__Employed), predict(pr outcome(1))
1._predict
2._predict
            : Pr(empstat==2__Unemployed), predict(pr outcome(2))
            : Pr(empstat==3__Not_in_labor_force), predict(pr outcome(3))
3._predict
            : 0.female
                 1.edu
                                    .1029445 (mean)
                 2.edu
                                   .2354108 (mean)
                 3.edu
                                     .2854081 (mean)
                 4.edu
                                     .235196 (mean)
                 5.edu
                                = .1410406 (mean)
                 female
                                           0
                                = .6620612 (mean)
                 0.minority
                                    .3379388 (mean)
                 1.minority
                 age
                                = 39.54567 (mean)
                 age2
                                = 1572.055 (mean)
              1.female
                 1.edu
                                   .0808115 (mean)
                 2.edu
                                =
                                    .2108511 (mean)
                                     .3082787 (mean)
                 3.edu
                 4.edu
                                    .2521074 (mean)
                 5.edu
                                =
                                   .1479512 (mean)
                 female
                 0.minority
                               = .6580153 (mean)
                 1.minority
                                     .3419847 (mean)
                                     39.55048 (mean)
                 age
                                = 1572.278 (mean)
                 age2
                              Delta-method
                                                            [95% Conf. Interval]
                        Margin Std. Err. z
                                                   P>|z|
_predict#female#edu |
            101 |
                     .8248057 .0109713 75.18
                                                   0.000
                                                            .8033024
                                                                        .846309
                      .8806449
            102 |
                                .006263 140.61
                                                   0.000
                                                                       .8929202
                                                            .8683697
            1 0 3
                      .9086188
                                .0049485
                                         183.62
                                                  0.000
                                                            .8989199
                                                                        .9183176
            1 0 4
                                                   0.000
                      .9369143
                                .0036388
                                         257.48
                                                            .9297823
                                                                      .9440463
            105 |
                      .9628471
                                .0031045
                                          310.15
                                                   0.000
                                                            .9567624
                                                                      .9689318
            1 1 1 |
                                .0183012
                                           28.06
                                                   0.000
                       .513562
                                                            .4776924
                                                                        .5494316
            1 1 2
                  - 1
                      .6537221
                                .0110446
                                           59.19
                                                   0.000
                                                            .6320751
                                                                        .6753692
                                .0083771
            1 1 3 |
                      .731767
                                           87.35
                                                   0.000
                                                            .7153481
                                                                        .7481858
            1 1 4 |
                      .7613286
                                .008998 84.61
                                                   0.000
                                                            .7436928
                                                                       .7789644
                                           98.04
                                                   0.000
            1 1 5
                      .8543542
                                .0087141
                                                            .8372749
                                                                        .8714334
            2 0 1
                      .0849345
                                .0085949
                                            9.88
                                                  0.000
                                                            .0680888
                                                                        .1017801
            202 |
                      .0684232
                                .005294 12.92 0.000
                                                            .0580472
                                                                      .0787992
            203 |
                      .0562547
                                .0043786 12.85
                                                   0.000
                                                            .0476728
                                                                       .0648367
            2 0 4 |
                      .0285931
                                .0027867
                                           10.26
                                                   0.000
                                                            .0231313
                                                                        .0340549
                                                            .013191
            2 0 5 l
                      .0182583
                                .0025854
                                           7.06
                                                   0.000
                                                                        .0233255
            2 1 1 |
                      .0661529
                                .0074037
                                            8.94
                                                   0.000
                                                             .051642
                                                                        .0806639
            2 1 2 |
                       .063536
                                .0049815
                                         12.75
                                                   0.000
                                                            .0537725
                                                                        .0732995
            2 1 3 |
                      .0566727
                                 .0041721
                                            13.58
                                                   0.000
                                                            .0484956
                                                                        .0648499
```

```
2 1 4 | .0290641 .002782 10.45 0.000
                                                      .0236115
                                                               .0345168
           2 1 5 | .0202658 .0029099 6.96
                                              0.000
                                                      .0145626 .0259691
                                                       .0758734
           3 0 1 |
                   .0902598 .0073401 12.30
                                              0.000
                                                                 .1046462
                             .0035917 14.18
           302 |
                   .0509319
                                              0.000
                                                      .0438923
                                                                 .0579714
           3 0 3 |
                  .0351265
                              .00243 14.46
                                              0.000
                                                      .0303637
                                                                .0398893
                   .0344926
           3 0 4 |
                            .0023514 14.67
                                              0.000
                                                       .029884
                                                                 .0391012
           3 0 5 |
                    .0188947
                             .0017217
                                        10.97
                                              0.000
                                                       .0155203
                                                                 .0222691
           3 1 1 |
                  .4202851 .0190743
                                       22.03 0.000
                                                      .3829002 .4576699
           3 1 2 |
                  .2827419 .0108342 26.10 0.000
                                                      .2615073 .3039765
                             .0078637 26.90
                  .2115603
                                              0.000
           3 1 3 |
                                                       . 1961478
                                                                 .2269728
                             .0088578 23.66
           3 1 4 |
                    .2096072
                                              0.000
                                                      .1922463
                                                                 .2269682
                     .12538 .0084167
           3 1 5
                                       14.90
                                              0.000
                                                       .1088836
                                                                 .1418765
. margins i.edu, dydx(i.female) atmeans
Conditional marginal effects
                                        Number of obs =
                                                             22,256
Model VCE : Robust
dy/dx w.r.t. : 1.female
          : Pr(empstat==1__Employed), predict(pr outcome(1))
1._predict
2._predict : Pr(empstat==2__Unemployed), predict(pr outcome(2))
3._predict : Pr(empstat==3__Not_in_labor_force), predict(pr outcome(3))
           : 1.edu = .0918028 (mean)
at
            2.edu
                         = .2230475 (mean)
                         = .2969211 (mean)
            3.edu
                         = .2437092 (mean)
            4.edu
                        = .1445194 (mean)
            5.edu
                        = .4966023 (mean)
            0.female
                         = .5033977 (mean)
            1.female
            0.minority = .6600245 (mean)
1.minority = .3399755 (mean)
                          = 39.54809 (mean)
            age
                         = 1572.167 (mean)
            age2
               ______
                      Delta-method
                dy/dx Std. Err. z P>|z| [95% Conf. Interval]
0.female
         | (base outcome)
          1.female
_predict#edu |
      1 1 | -.3111994 .0140035 -22.22 0.000
                                                -.3386457
                                                          -.2837532
      1 2 | -.2268817
                       .0098033
                                 -23.14 0.000
                                                -.2460957
                                                           -.2076676
      1 3 | -.1768153
                      .0077511 -22.81 0.000
                                                -.1920073
                                                         -.1616233
      1 4 | -.1755627
                      .0080303 -21.86 0.000
                                                -.1913018 -.1598236
      1 5 | -.1084769
                       .0072672 -14.93 0.000
                                                          -.0942334
                                                -.1227204
      2 1 | -.0188237
                       .0058193
                                -3.23 0.001
                                                -.0302294
                                                          -.0074181
      2 2 | -.0049249
                       .0049082 -1.00 0.316
                                                -.0145447
                                                         .0046949
                      .0042242 0.09 0.927
      2 3 | .0003853
                                                -.0078941
                                                         .0086646
                                  0.20
      24 |
                                        0.838
              .0004538
                       .0022263
                                                -.0039097
                                                           .0048173
      2 5 I
                                  1.30
              .0019959
                       .0015355
                                        0.194
                                                -.0010137
                                                           .0050054
```

3 1	.3300231	.0147675	22.35	0.000	.3010794	.3589669
3 2	.2318066	.0094055	24.65	0.000	.2133722	.2502409
3 3	.17643	.0070254	25.11	0.000	.1626606	.1901995
3 4	.1751088	.0079644	21.99	0.000	.1594989	.1907188
3 5	.1064811	.0072558	14.68	0.000	.09226	.1207022

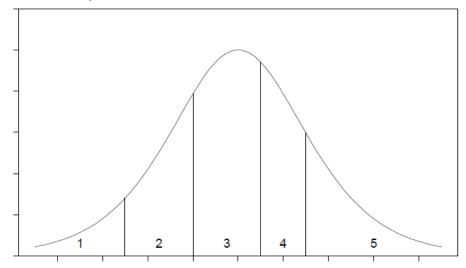
Note: dy/dx for factor levels is the discrete change from the base level.

If you would like to change the base outcome to (2) Unemployed, estimate the following command.

mlogit empstat i.edu female minority age age2 mar[pw=perwt], baseoutcome(2)

#### Ordinal Logit or Ordered Logit

- 1. Ordered Logit models (ologit) can be used when your outcome variable is ordered (for example, high=3, medium=2, and low=1; for another example, strongly agree=4, somewhat agree=3, somewhat disagree=2, strongly disagree=1).
- 2. These models can also be interpreted in terms of a latent variable. Specifically, suppose that the manifest response  $Y_i$  results from grouping an underlying continuous variable  $Y_i^*$  using cut-points  $\theta_1 < \theta_2 ... < \theta_{J-1}$ , so that  $Y_i$  takes the value 1 if  $Y_i^*$  is below  $\theta_1$ , the value 2 if  $Y_i^*$  is between  $\theta_1$  and  $\theta_2$ , and so on, taking the value J if  $Y_i^*$  is above  $\theta_{J-1}$ .



- 3. Thus, Ordered Logit estimates the effect of independent variables as a function of cumulative probabilities. Because it is a cumulative probability function, Ordered Logit estimates one coefficient for each independent variable and compute
- 4. The cumulative probability  $C_{ij}$  for individual i up to response level j,

$$C_{ij} = Pr(y_j \le j) = Pr(y_i \le j | x) = Pr(x / \beta + \epsilon \le \theta_j),$$

Rearrange terms, we find that:

$$C_{ij} = Pr(\epsilon \le \theta_j - x\prime\beta) = F(\theta_j - x\prime\beta)$$

Thus, the probability of  $y_i = j$  given x is,

$$Pr(y_i = j | X_i) = \begin{cases} F(\theta_1 - x'\beta), & j = 1\\ F(\theta_j - x'\beta) - F(\theta_{j-1} - x'\beta), & 1 < j \le J - 1\\ 1 - F(\theta_{J-1} - x'\beta), & j = J \end{cases}$$

In Stata,  $\theta_i$  are cut points. Note that only one set of  $\beta$ 's are estimated.

5. The probability of  $C_{ij}$  can be estimated as follows:

$$C_{ij} = \frac{\exp(\theta_j + x'\beta)}{1 + \exp(\theta_j + x'\beta)}$$

6. Interpretation: Conditional on the other covariates, the odds that  $Y_j$  is less than or equal to a given level j versus greater than j is estimated to be  $\exp(\beta)$  times greater as x increases by 1 unit.

Ordered logit: dep variable = Subjective Well-being (5= high, 1=low)

```
. ologit SWB SSS c.age##c.age female yrsch [pw=weight]
Iteration 0:
             log pseudolikelihood = -13331.542
Iteration 1:
             log pseudolikelihood = -12972.818
Iteration 2: log pseudolikelihood = -12969.055
Iteration 3:
             log pseudolikelihood = -12969.053
Iteration 4: log pseudolikelihood = -12969.053
Ordered logistic regression
                                          Number of obs = 11,001
                                          Wald chi2(5) = 535.31
                                          Prob > chi2
                                                               0.0000
Log pseudolikelihood = -12969.053
                                          Pseudo R2
                                                               0.0272
                         Robust
       SWB | Coef. Std. Err. z P>|z| [95% Conf. Interval]
                                   21.53 0.000 .2622015 .3147244
        SSS |
              .288463 .0133989
        age | -.0197745 .0068535 -2.89 0.004
                                                  -.0332072 -.0063418
 c.age#c.age |
                                   2.50 0.013
               .0001688 .0000676
                                                  .0000362
                                                           .0003013
     female | -.0299368 .0400124
                                   -0.75 0.454
                                                  -.1083596
                                                              .048486
      yrsch | .0028191 .0050849 0.55 0.579
                                                  -.0071471
                                                              .0127854
      /cut1 | -3.373698 .2013347
                                                  -3.768307
                                                            -2.979089
      /cut2 | -1.68202 .1899033
                                                  -2.054223 -1.309816
      /cut3 | -.2022073 .1863877
                                                  -.5675205
                                                             .1631058
      /cut4 |
               2.366788
                        .1875696
                                                   1.999158
                                                              2.734417
```

Estimating the probabilities of 5 Likert-scale outcomes.

```
. margins

Predictive margins

Mumber of obs = 11,001

Model VCE : Robust
```

Comparing the probabilities of 5 Likert-scale outcomes by Subjective Social Standing

```
. margins, at(SSS=(1(3)10))
Predictive margins
                                     Number of obs =
                                                        11,001
Model VCE : Robust
1._predict : Pr(SWB==1), predict(pr outcome(1))
2._predict : Pr(SWB==2), predict(pr outcome(2))
3._predict : Pr(SWB==3), predict(pr outcome(3))
4._predict : Pr(SWB==4), predict(pr outcome(4))
5._predict
         : Pr(SWB==5), predict(pr outcome(5))
1._at : SSS
2._at : SSS
3._at
         : SSS
                                 7
4._at
         : SSS
                                10
            Delta-method
             Margin Std. Err. z P>|z| [95% Conf. Interval]
      _predict#_at |
```

.003817	.0024667	0.000	9.12	.0003445	.0031418	ı	1 4
.1599006	.1319598	0.000	20.47	.0071279	.1459302	1	2 1
.0759179	.0651958	0.000	25.79	.0027353	.0705568		2 2
.034944	.0283005	0.000	18.66	.0016948	.0316223		2 3
.0159872	.0113695	0.000	11.61	.001178	.0136784		2 4
.3292431	.3002068	0.000	42.49	.0074074	.3147249		3 1
.2180494	.2008473	0.000	47.73	.0043884	.2094484		3 2
.1204891	.1041636	0.000	26.97	.0041648	.1123264		3 3
.060702	.0453944	0.000	13.58	.0039051	.0530482		3 4
.4464227	.4091147	0.000	44.95	.0095175	.4277687		4 1
.5591537	.538557	0.000	104.46	.0052543	.5488553		4 2
.5588242	.5360859	0.000	94.38	.0058007	.5474551		4 3
.4512842	.3978223	0.000	31.13	.0136385	.4245533		4 4
.0787107	.0633448	0.000	18.12	.0039199	.0710278		5 1
.1612852	.1460356	0.000	39.50	.0038903	.1536604		5 2
.3170897	.2852381	0.000	37.06	.0081255	.3011639	-	5 3
.5412934	.4698633	0.000	27.75	.0182223	.5055784		5 4