Week 11. Panel Models 2

Types of Panel Models

- FEM: Fixed Effects Model

$$y_{it} = \alpha + \sum \beta_k X_{ikt} + u_i + e_{it}$$

or,

$$y_{it} = \alpha_i + \sum \beta_k X_{ikt} + e_{it}$$

- FDM: First Difference Model

$$\Delta y_i = \alpha + \sum \beta_k \Delta X_{ik} + e_i$$

- REM: Random Effects Model

$$(y_{it} - \theta \bar{y}_i) = \alpha(1-\theta) + \sum \beta_k (X_{ikt} - \theta \bar{X}_{ik}) + (e_{it} - \theta \bar{e}_i)$$

- CRE (The Correlated Random Effects Model) = Hybrid Model

$$y_{it} = \alpha + \sum \beta_k X_{ikt} + \sum \gamma_l \bar{X}_{ik} + \sum \delta_m Z_i + \epsilon_i + e_{it}$$

where Z_i is a vector of time-invariant covariates,

$$u_i = \sum \gamma_l \bar{X}_{ik} + \sum \delta_m Z_i + \epsilon_i$$

- RCM: Random Coefficients Model (or Mixed Model, or Linear Mixed Model or Multilevel Growth Curve Model)
- There are many other panel models which we will not discuss.

FEM: Interaction of Time-varying covariates $\times$ Time-invariant covariates and Interaction of Time-varying covariates $\times$ Time-varying covariates

- Interaction of Time-varying covariates(X_1) $\times$ Time-invariant covariates(X_2): As X_1 changes by 1-unit, how much y will change differently by X_2.
- Interaction of Time-varying covariates(X_1) $\times$ Time-varying covariates(X_3): As X_1 increases by 1-unit, how much y will change differently when X_3 increases by 1-unit.
- In the below example,
  - `SSS`: Subjective social standing (1: lowest, 6: highest)
  - `highim2pop`: % high-status immigrants by region
  - `lowercl`: Low-class natives, time-invariant class status
  - `highercl`: High-class natives, time-invariant class status
  - `rank1`: Relative income rank by year, time-varying variable.
  - `marst`, `edu`: marital status, and education. Both are time-varying.

```
Fixed-effects (within) regression               Number of obs     =    155,306
Group variable: pid                             Number of groups  =     28,241

R-sq:                                           Obs per group:
     within  = 0.0313                                         min =          1
     between = 0.1736                                         avg =        5.5
     overall = 0.1417                                         max =         10

                                       (Std. Err. adjusted for 28,241 clusters in pid)
--------------------------------------------------------------------------------------
                     |               Robust
                 SSS |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------------+----------------------------------------------------------------
           highim2pop |  -.0929533   .0467272    -1.99   0.047    -.1845409   -.0013656
                     |
          1.lowercl |          0  (omitted)
lowercl#c.highim2pop |
                  1  |    .021822   .0987126     0.22   0.825    -.1716593    .2153034
                     |
          1.uppercl |          0  (omitted)
uppercl#c.highim2pop |
                  1  |   .1930963   .0809319     2.39   0.017     .0344659    .3517267
                     |
               rank1 |   .0040681   .0001519    26.79   0.000     .0037704    .0043658
c.highim2pop#c.rank1 |   .0007305   .0005505     1.33   0.184    -.0003484    .0018094
                     |
                 age |  -.0183471   .0028889    -6.35   0.000    -.0240094   -.0126848
         c.age#c.age |   .0000653   .0000256     2.55   0.011     .0000151    .0001154
                     |
              female |          0  (omitted)
                     |
               marst |
                  2  |   .0652948   .0210354     3.10   0.002     .0240644    .1065252
                  3  |  -.1318094   .0515807    -2.56   0.011      -.23291   -.0307088
                  4  |  -.0898101   .0383994    -2.34   0.019    -.1650748   -.0145455
                  5  |  -.0009762   .0346178    -0.03   0.978    -.0688289    .0668764
                     |
              famnum |   -.011906    .004652    -2.56   0.010    -.0210241   -.0027879
                     |
                 edu |
                  2  |  -.0502888   .0191458    -2.63   0.009    -.0878155   -.0127621
                  3  |    -.09479    .019817    -4.78   0.000    -.1336322   -.0559478
                  4  |  -.1348723   .0317038    -4.25   0.000    -.1970132   -.0727313
                  5  |  -.0671691   .0547998    -1.23   0.220    -.1745793    .0402411
```

Random Coefficients Model: RCM

- In a Nutshell: In OLS, we have one intercept and one coefficient for each variable (e.g., time). In RCM, this assumption can be relaxed by allowing the intercept and slope to vary across individuals and be predicted by other covariates. That is, OLS models estimate means, while RCM models estimate both means and variance components.

- Random Coefficients Model = a multi-level model.
  Here the 1st level is individual and the 2nd level is individual-time.

- Key Background Idea: Independence of the observations is a key assumption of many standard statistical methods including OLS. Common examples of data structures that do not fit into such a framework arise in longitudinal analysis, in which observations are made on subjects at subject-specific sequences of time points, and in studies that involve subjects (units) occurring naturally in clusters (or groups), such as individuals within families, school children within classrooms, employees within companies, and the like. The assumption of independence of the observational units is not tenable when observations within a cluster tend to be more similar than observations in general. REM is a relaxation of this assumption of independence of the observational units.

- In OLS,
  $$y_{it} = \alpha + \sum \beta_k X_{ikt} + e_{it}$$
  If there is only one independent variable, T in OLS,
  $$y_{it} = \alpha + \beta T_{it} + e_{it}$$
  We have usual assumptions such as normality, independence and equal variance (homoscedasticity) of the deviations e_{it}. In this case, $e_{it} \sim N(0, \sigma^2)$, i.i.d., implies that the regressions within the clusters/individuals/groups i have a common vector of coefficients β.

- We only have one intercept and one coefficient. Thus, they are fixed across observations.

- This restriction (= coefficients estimated are fixed) can be relaxed by allowing the regressions to differ in their intercepts and slopes. In REM, we model how systematically intercept and coefficients across clusters/groups/individuals.

- $$y_{it} = (\alpha + b_{0i}) + (\beta + b_{1i})T_{it} + \varepsilon_{it}$$
  Here, α and β are fixed effects and b_{0i} and b_{1i} are random effects.
  $$\begin{aligned} b_{0i} &= b_{00} + u_{0i} \\ b_{1i} &= b_{10} + u_{1i} \end{aligned} \tag{1}$$

- Now we have three error components: ε_{it}, u_{0i}, and u_{1i}. Recall that, in OLS, we only have one error component ε_{it} which is normally distributed with mean 0, $\varepsilon_{it} \sim N(0, \sigma^2_{\varepsilon})$. Like OLS, we assume that ε_{it} is normally distributed with mean 0 and variance of σ^2_{ε}.

- The other two error components are also assumed to be normally distributed with mean 0. That is, $u_{0i} \sim N(0, \sigma^2_{u_0})$ and $u_{1i} \sim N(0, \sigma^2_{u_1})$

- The main concern is how these three error components (= ε_{it}, u_{0i}, and u_{1i}) are correlated with each other.

- For all regression models, ε_{it} is assumed to be uncorrelated with other components. Otherwise, the coefficients estimated are biased. Thus, the error structure of the other two errors (= the variance-covariance matrix of u_{0i} and u_{1i}) is uncorrelated with ε_{it}. Let's say the variance-covariance matrix of u_{0i} and u_{1i} is G. The covariance between G and ε_{it} is assumed to be zero. Then,

$$Var \begin{bmatrix} u_{0i}, u_{1i} \\ \varepsilon_{it} \end{bmatrix} = \begin{bmatrix} G & 0 \\ 0 & \sigma^2_{\varepsilon_{it}} \end{bmatrix}$$

- For u_{0i} and u_{1i}, it is assumed that u_{0i} and u_{1i} are jointly normally distributed with mean 0, the variances of $\sigma^2_{u_0}$ and $\sigma^2_{u_1}$, and the covariance of $\rho(u_0, u_1)$. That is,

$$Var \begin{bmatrix} u_{0i} \\ u_{1i} \end{bmatrix} = \begin{bmatrix} \sigma^2_{u_0} & \rho_{u_0,u_1} \\ \rho_{u_0,u_1} & \sigma^2_{u_1} \end{bmatrix} \sim N \begin{bmatrix} \sigma^2_{u_0} & \rho_{u_0,u_1} \\ \rho_{u_0,u_1} & \sigma^2_{u_1} \end{bmatrix}$$

  Sometimes it is assumed that $\rho_{u_0,u_1} = 0$.

- Now we add independent variables to account for the random effects.

$$\begin{aligned} y_{it} &= (\alpha + b_{0i}) + (\beta + b_{1i})T_{it} + \varepsilon_{it} \\ &\quad \text{Here, } \alpha \text{ and } \beta \text{ are fixed effects and } b_{0i} \text{ and } b_{1i} \text{ are random effects.} \\ b_{0i} &= b_{00} + \sum b_{0k}X_{ikt} + u_{0i} \\ b_{1i} &= b_{10} + \sum b_{1k}X_{ikt} + u_{1i} \end{aligned} \tag{2}$$

- Combining the fixed and random components, equation (2) becomes equation (3):

$$\begin{aligned} y_{it} &= (\alpha + b_{00} + \sum b_{0k}X_{ikt} + u_{0i}) + (\beta + b_{10} + \sum b_{1k}X_{ikt} + u_{1i})T_{it} + \varepsilon_{it} \\ &= (\alpha + b_{00}) + (\beta + b_{10})T_{it} + \sum b_{0k}X_{ikt} + \sum b_{1k}(X_{ikt} \times T_{it}) \\ &+ u_{0i} + u_{1i}T_{it} + \varepsilon_{it} \\ &= \alpha' + \beta' T_{it} + \sum b_{0k}X_{ikt} + \sum b_{1k}(X_{ikt} \times T_{it}) + (u_{0i} + u_{1i}T_{it} + \varepsilon_{it}) \end{aligned} \tag{3}$$

- That is, T_{it}, X_{ikt}, and $(X_{ikt} \times T_{it})$ are a set of covariates you need to add in your model.

RCM: Random Intercept without Other Covariates and Fixed Slope

$$y_{it} = (\alpha + b_{0i}) + \beta T_{it} + \varepsilon_{it}$$
$$b_{0i} = b_{00} + u_{0i}$$

$$Var \begin{bmatrix} u_{0i} \\ \epsilon_{it} \end{bmatrix} = \sim N \begin{bmatrix} \sigma^2_{u_0} & 0 \\ 0 & \sigma^2_{\epsilon_{it}} \end{bmatrix}$$

```
. mixed lnwage time || ind:

Performing EM optimization:
Performing gradient-based optimization:
Computing standard errors:

Mixed-effects ML regression                     Number of obs     =        840
Group variable: ind                             Number of groups  =         42

                                                Obs per group:
                                                              min =         20
                                                              avg =       20.0
                                                              max =         20

                                                Wald chi2(1)      =     369.69
Log likelihood =  1273.1363                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        time |   .0051957   .0002702    19.23   0.000     .0046661    .0057254
       _cons |   2.523098   .0406276    62.10   0.000      2.44347    2.602727
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
ind: Identity                |
                  var(_cons) |   .0689465   .0150676      .0449252    .1058118
-----------------------------+------------------------------------------------
               var(Residual) |   .0020395   .0001021      .0018489    .0022498
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 2708.06       Prob >= chibar2 = 0.0000
```

RCM: Random Intercept and Random Slope without Other Covariates

$$y_{it} = (\alpha + b_{0i}) + (\beta + b_{1i})T_{it} + \varepsilon_{it}$$
$$b_{0i} = b_{00} + u_{0i}$$
$$b_{1i} = b_{10} + u_{1i}$$

$$Var \begin{bmatrix} u_{0i} \\ u_{1i} \end{bmatrix} =\sim N \begin{bmatrix} \sigma^2_{u_0} & 0 \\ 0 & \sigma^2_{u_1} \end{bmatrix}$$

```
. mixed lnwage time || ind: time

Performing EM optimization:
Performing gradient-based optimization:
Computing standard errors:

Mixed-effects ML regression                     Number of obs     =        840
Group variable: ind                             Number of groups  =         42

                                                Obs per group:
                                                              min =         20
                                                              avg =       20.0
                                                              max =         20

                                                Wald chi2(1)      =      37.83
Log likelihood =  1477.8734                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        time |   .0051957   .0008447     6.15   0.000       .00354    .0068514
       _cons |   2.523098   .0455192    55.43   0.000     2.433882    2.612314
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
ind: Independent             |
                  var(time) |   .0000284   6.54e-06      .0000181    .0000446
                 var(_cons) |   .0868322   .0189902      .0565617    .1333027
-----------------------------+------------------------------------------------
              var(Residual) |   .0010321   .0000531      .0009331    .0011416
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 3117.53              Prob > chi2 = 0.0000
```

RCM: Random Intercept net of Other Covariates and Fixed Slope

$$y_{it} = (\alpha + b_{0i}) + (\beta + b_{1i})T_{it} + \varepsilon_{it}$$
$$b_{0i} = b_{00} + \sum b_{0k} X_{ikt} + u_{0i}$$

```
. mixed lnwage time union part || ind:

Performing EM optimization:
Performing gradient-based optimization:
Computing standard errors:

Mixed-effects ML regression                     Number of obs     =        840
Group variable: ind                             Number of groups  =         42

                                                Obs per group:
                                                              min =         20
                                                              avg =       20.0
                                                              max =         20

                                                Wald chi2(3)      =     830.51
Log likelihood =  1409.9854                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        time |   .0066589   .0004251    15.66   0.000     .0058257    .0074922
       union |   .5420962   .0553771     9.79   0.000     .4335592    .6506333
        part |  -.7703482   .0766178   -10.05   0.000    -.9205164   -.6201801
       _cons |   2.486188   .0344645    72.14   0.000     2.418639    2.553738
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
ind: Identity                |
                  var(_cons) |   .0354159    .007816      .0229797    .0545823
-----------------------------+------------------------------------------------
               var(Residual) |   .0014989   .0000751      .0013588    .0016536
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 2313.07       Prob >= chibar2 = 0.0000
```

RCM: Random Intercept net of Other Covariates and Random Slope

$$y_{it} = (\alpha + b_{0i}) + (\beta + b_{1i})T_{it} + \varepsilon_{it}$$
$$b_{0i} = b_{00} + \sum b_{0k}X_{ikt} + u_{0i}$$
$$b_{1i} = b_{10} + u_{1i}$$

$$Var \begin{bmatrix} u_{0i} \\ u_{1i} \end{bmatrix} = \sim N \begin{bmatrix} \sigma^2_{u_0} & 0 \\ 0 & \sigma^2_{u_1} \end{bmatrix}$$

```
. mixed lnwage time union part || ind: time

Performing EM optimization:
Performing gradient-based optimization:
Computing standard errors:

Mixed-effects ML regression                     Number of obs     =        840
Group variable: ind                             Number of groups  =         42

                                                Obs per group:
                                                              min =         20
                                                              avg =       20.0
                                                              max =         20

                                                Wald chi2(3)      =     156.00
Log likelihood =  1511.0846                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        time |   .0056969   .0007439     7.66   0.000     .0042389    .0071549
       union |     .31924   .0676072     4.72   0.000     .1867322    .4517477
        part |  -.6651201   .0943489    -7.05   0.000    -.8500404   -.4801997
       _cons |   2.525942   .0390991    64.60   0.000     2.449309    2.602575
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
ind: Independent             |
                  var(time) |   .0000148   3.66e-06      9.08e-06     .000024
                 var(_cons) |   .0459613    .010538      .0293244    .0720371
-----------------------------+------------------------------------------------
              var(Residual) |   .0010129   .0000526       .000915    .0011214
------------------------------------------------------------------------------
LR test vs. linear model: chi2(2) = 2515.26              Prob > chi2 = 0.0000
```

RCM: Random Intercept net of Other Covariates and Random Slope, Unstructured

$$y_{it} = (\alpha + b_{0i}) + (\beta + b_{1i})T_{it} + \varepsilon_{it}$$
$$b_{0i} = b_{00} + \sum b_{0k}X_{ikt} + u_{0i}$$
$$b_{1i} = b_{10} + u_{1i}$$

$$Var\begin{bmatrix} u_{0i} \\ u_{1i} \end{bmatrix} = \sim N\begin{bmatrix} \sigma^2_{u_0} & \rho(u_0,u_1) \\ \rho(u_0,u_1) & \sigma^2_{u_1} \end{bmatrix}$$

```
. mixed lnwage time union part || ind: time, covariance(unstructured)

Performing EM optimization:
Performing gradient-based optimization:
Computing standard errors:

Mixed-effects ML regression                     Number of obs     =        840
Group variable: ind                             Number of groups  =         42

                                                Obs per group:
                                                              min =         20
                                                              avg =       20.0
                                                              max =         20

                                                Wald chi2(3)      =     126.99
Log likelihood =  1516.0186                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        time |   .0056362   .0007628     7.39   0.000     .0041412    .0071313
       union |   .2843386   .0689311     4.12   0.000     .1492361     .419441
        part |  -.5958343   .0982291    -6.07   0.000    -.7883599   -.4033088
       _cons |    2.52603    .040325    62.64   0.000     2.446994    2.605066
------------------------------------------------------------------------------


------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
ind: Unstructured            |
                   var(time) |   .0000158   3.95e-06      9.72e-06    .0000258
                  var(_cons) |   .0496985   .0115182      .0315549    .0782744
             cov(time,_cons) |   -.000438   .0001722     -.0007756   -.0001005
-----------------------------+------------------------------------------------
               var(Residual) |   .0010069    .000052      .0009099    .0011143
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 2525.13              Prob > chi2 = 0.0000
```

RCM: Random Intercept net of Other Covariates and Random Slope net of Other Covariates, Unstructured

$$y_{it} = (\alpha + b_{0i}) + (\beta + b_{1i})T_{it} + \varepsilon_{it}$$
$$b_{0i} = b_{00} + \sum b_{0k}X_{ikt} + u_{0i}$$
$$b_{1i} = b_{10} + \sum b_{1k}X_{ikt} + u_{1i}$$

$$Var \begin{bmatrix} u_{0i} \\ u_{1i} \end{bmatrix} =\sim N \begin{bmatrix} \sigma^2_{u_0} & \rho(u_0, u_1) \\ \rho(u_0, u_1) & \sigma^2_{u_1} \end{bmatrix}$$

```
. mixed lnwage time union part c.time##c.union c.time##c.part || ind: time, covariance(unstructured)

Performing EM optimization:
Performing gradient-based optimization:
Computing standard errors:

Mixed-effects ML regression                     Number of obs     =        840
Group variable: ind                             Number of groups  =         42

                                                Obs per group:
                                                              min =         20
                                                              avg =       20.0
                                                              max =         20

                                                Wald chi2(5)      =     248.34
Log likelihood =  1581.4253                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        time |   .0176998   .0014519    12.19   0.000     .0148543    .0205454
       union |   .4372819   .0668816     6.54   0.000     .3061963    .5683675
        part |  -.1465969   .1114184    -1.32   0.188     -.364973    .0717793
        time |          0  (omitted)
       union |          0  (omitted)
             |
c.time#c.union |  -.0483779   .0041552   -11.64   0.000     -.056522   -.0402338
             |
        time |          0  (omitted)
        part |          0  (omitted)
             |
 c.time#c.part |  -.0451379   .0070325    -6.42   0.000    -.0589214   -.0313545
             |
       _cons |   2.455194   .0433314    56.66   0.000     2.370266    2.540122
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
```

```
-----------------------------+------------------------------------------------
ind: Unstructured            |
                   var(time) |   .0000391   .0000131      .0000202    .0000754
                  var(_cons) |   .0605239    .014725      .0375695     .097503
            cov(time,_cons) |  -.0006508   .0003655     -.0013672    .0000656
-----------------------------+------------------------------------------------
               var(Residual) |   .0007963   .0000425      .0007173    .0008841
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 2654.55              Prob > chi2 = 0.0000
```

RCM vs REM

- If we assume only intercepts differ across clusters/groups/individuals and the difference is random, then,
$$y_{it} = (\alpha + b_{0i}) + \beta T_{it} + \varepsilon_{it}$$
This is REM. Thus, REM is a special case of RCM. If you estimate a simple random-intercept RCM, you will get the identical results with REM with the option of MLE (maximum likelihood estimation).

```
. xtset ind time

* ========
* REM
* ========
. xtreg lnwage time union part, re mle

Random-effects ML regression                    Number of obs      =       840
Group variable: ind                             Number of groups   =        42

Random effects u_i ~ Gaussian                   Obs per group:
                                                              min =        20
                                                              avg =      20.0
                                                              max =        20

                                                LR chi2(3)         =    577.48
Log likelihood  =  1409.9854                    Prob > chi2        =    0.0000

------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        time |   .0066589   .0004252    15.66   0.000     .0058255    .0074924
       union |   .5420962   .0553895     9.79   0.000     .4335349    .6506576
        part |  -.7703483   .0771461    -9.99   0.000    -.9215517   -.6191448
       _cons |   2.486188   .0344729    72.12   0.000     2.418623    2.553754
-------------+----------------------------------------------------------------
    /sigma_u |   .1881911    .020766                      .1515908    .2336282
    /sigma_e |   .0387161   .0009696                      .0368617    .0406638
         rho |   .9593946   .0088249                      .9388778    .9739306
------------------------------------------------------------------------------
LR test of sigma_u=0: chibar2(01) = 2313.07           Prob >= chibar2 = 0.000


* ========
* RCM with two-level random intercept
* ========
. mixed lnwage time union part || ind:

Performing EM optimization:
Performing gradient-based optimization:
Computing standard errors:

Mixed-effects ML regression                     Number of obs      =       840
Group variable: ind                             Number of groups   =        42
```

```
                                                Obs per group:
                                                              min =         20
                                                              avg =       20.0
                                                              max =         20

                                                Wald chi2(3)      =     830.51
Log likelihood =  1409.9854                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
      lnwage |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        time |   .0066589   .0004251    15.66   0.000     .0058257    .0074922
       union |   .5420962   .0553771     9.79   0.000     .4335592    .6506333
        part |  -.7703482   .0766178   -10.05   0.000    -.9205164   -.6201801
       _cons |   2.486188   .0344645    72.14   0.000     2.418639    2.553738
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects Parameters  |   Estimate   Std. Err.     [95% Conf. Interval]
-----------------------------+------------------------------------------------
ind: Identity                |
                  var(_cons) |   .0354159    .007816      .0229797    .0545823
-----------------------------+------------------------------------------------
               var(Residual) |   .0014989   .0000751      .0013588    .0016536
------------------------------------------------------------------------------
LR test vs. linear model: chibar2(01) = 2313.07       Prob >= chibar2 = 0.0000
```