FINAL PROJECT IDEAS: MATH 783

Key Dates:

- Tuesday, April 5: Project proposal due (1 page maximum).
- Thursday, April 21: Project outline due (5 pages maximum).
- Friday, May 13: Final project due (20 pages maximum).

Some problems to consider:

I. Reaction-Diffusion Equation (also known as Allen-Cahn, Chaffee-Infante, Nagumo)

\[ u_t = \varepsilon^2 \Delta u - f(u), \quad x \in \Omega \] (1)

where \( f(u) = u(u-1)(u-a) \) with appropriate/interesting boundary and initial conditions.

II. Cahn-Hilliard Equation

\[ u_t = -\Delta (\varepsilon^2 \Delta u - f(u)), \quad x \in \Omega \] (2)

where \( f(u) = u(u-1)(u-a) \) with appropriate/interesting boundary and initial conditions.

III. Kuramoto-Sivashinsky Equation

\[ u_t + \nu u_{xxxx} + u_{xx} + \frac{1}{2} (u_x)^2 = 0, \quad x \in (-L,L) \] (3)

with periodic boundary conditions.

IV. Complex Ginzburg-Landau Equation (also known as nonlinear Schrodinger)

\[ u_t - (\lambda + i \alpha) \Delta u + (\kappa + i \beta)|u|^2 u - \gamma u = 0, \quad x \in \Omega \] (4)

with homogeneous Dirichlet, Neumann, or Periodic boundary conditions.

V. 2-D Navier-Stokes Equation

\[ \rho_0 (u_t + (u \cdot \nabla) u) - \nu \Delta u + \nabla u = f, \quad \text{div}(u) = 0, \quad x \in \Omega \] (5)

with homogeneous Dirichlet or Periodic boundary conditions.

VI. Shallow Water Equations, see, e.g.,


VII. Barotropic Vorticity Equation, see, e.g.,

https://en.wikipedia.org/wiki/Barotropic_vorticity_equation

VIII. Other You may consider an alternative PDE that fits with your interests.

Requirements: The final project should result in a short paper (maximum 20 pages) that includes background on the PDE you consider, a description, analysis, and results of the numerical method(s) you use/develop. You may consider different methods and compare them, numerically investigate solution behavior as a function of a parameter in the problem. The basic components of a “good” numerical scheme are the time discretization, space discretization, method for solving nonlinear systems of equations, and method for solving linear systems of equations.

You may use previously developed software, e.g. codes in matlab, codes at www.netlib.org, but you should show sufficient knowledge in your paper/report of the software you employ.

For I, II, IV, and V you may consider \( \Omega = (0,1) \) in 1-D, \( \Omega = (0,1) \times (0,1) \) in 2-D, etc..