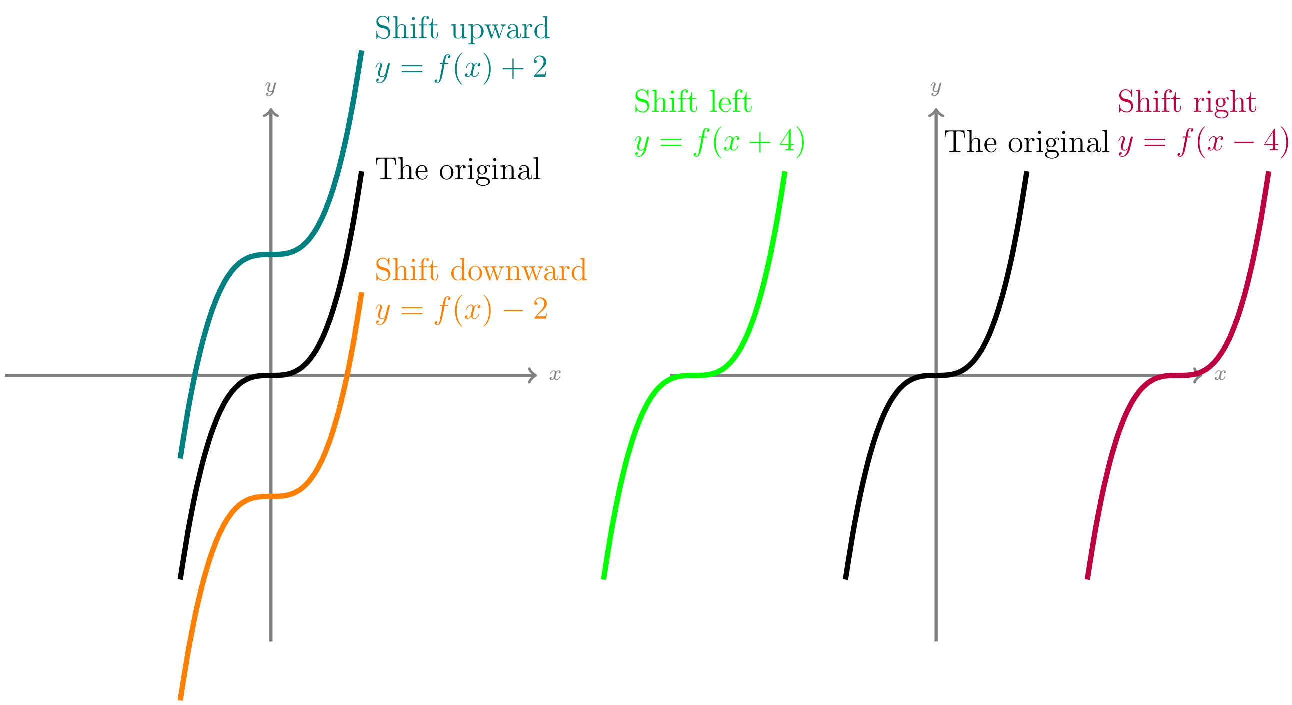
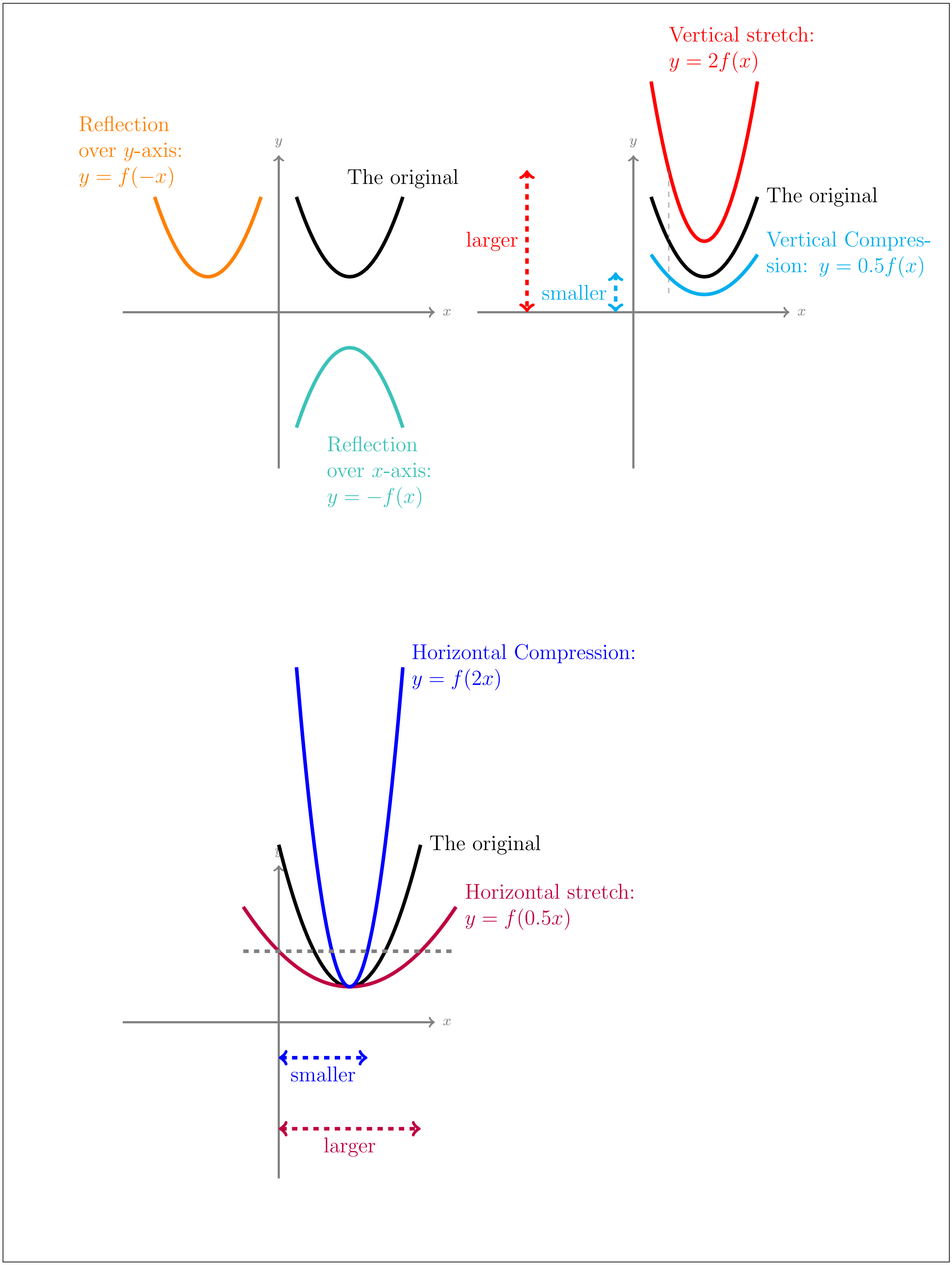
1.5: The Transformations of Functions

- Starting with the graph of the original function $y = f(x)$, a.k.a parent function, the following rules give the corresponding graph transformation.

Vertical shift by k units	$y = f(x) + k$	$k > 0$ upward shift ↑ $k < 0$ downward shift ↓
Horizontal shift by k units	$y = f(x + k)$	$k > 0$ shift to left ← $k < 0$ shift to right →
Reflection over x-axis	$y = -f(x)$	
Reflection over y-axis	$y = f(-x)$	
Reflection over Origin	$y = -f(-x)$	
Vertical Compression/stretch	$y = kf(x)$	$k > 1$ stretch $0 < k < 1$ Compression
Horizontal Compression/stretch	$y = f(kx)$	$k > 1$ Compression $0 < k < 1$ stretch




Reflection
over y-axis:
$y = f(-x)$
The original
Reflection
over x-axis:
$y = -f(x)$
Vertical stretch:
$y = 2f(x)$
larger
smaller
The original
Vertical Compression: $y = 0.5f(x)$
Horizontal Compression:
$y = f(2x)$
The original
Horizontal stretch:
$y = f(0.5x)$
smaller
larger
x
y

How to Perform the Horizontal Transformations

- In case of horizontal stretching/compression or shift, replace each input, for example x, by big parentheses, for example $\Big(ax\Big)$ or $\Big(x-a\Big)$.

Order of Transformations

Finding the transformations is part of the task. Please note that not all orders of the same transformations result in the same graph. The following is an easier option for ordering and is not unique.

- Horizontal shift. $(+a)$
- Horizontal (b) and/or vertical (c) stretch/compression.
- Reflections.
- Vertical shift.$(+d)$
- Then the function f after these transformations becomes $g(x) = cf(bx + a)) + d$.

Odd and Even Functions

- **Odd Functions:** If $f(x) = -f(-x)$, then f is odd.
- **Even Functions:** If $f(x) = f(-x)$, then f is even.
- General test for odd/even functions is to replace x, or the input variable, by $\Big(-x\Big)$ and simplify.If the function doesn't change, then it is even. If the function multiplies by a negative sign, then it is odd. Otherwise, the function is neither even nor odd, and you can find an input value that makes both of the above equations false.
- Odd functions are symmetric about the origin. Even functions are symmetric about y-axis.
- The name even matches the fact that all polynomial with terms of **only** even power are even. The name odd matches the fact that all polynomial with terms of **only** odd power are odd.
- Any constant c is considered to be the term cx^0 and 0 is an even power. That is, constant functions are even functions.
- We discuss trigonometric and other type of odd or even function later in the course.

1. Let $f(x) = \sqrt{x}$. Find a function $g(x)$ whose graph is the graph of $f(x)$ shifted left 3 units, vertically stretched by a factor of 4 and shifted down by 5 units, in that order. (Go through each transformation in order, and write down the functions.)

2. Consider the function $g(x) = 7|5x - 3| - 4$.

   (a) Identify the parent function and describe the transformation on g (shifts, stretches, etc).

   (b) Use this description to sketch a graph of g.

3. Consider the function $f(x) = -7(x-4)^2 + 3$.

   (a) Identify the parent function and describe the transformations on f (shifts, stretches, etc).

   (b) Use this description to sketch a graph of f.

Fun Project: Use the applet at https://ggbm.at/j7yuscgm to test different order of the transformation in part (a) of this problem. Which of the orders results in the same function?

4. Identify odd and even functions in the following list.

   (a) $f(x) = x^4 - x^2 - 3$

   (b) $g(t) = t^4 - 2t - 3$

   (c) $h(x) = x^3 - x - 1$

   (d) $i(y) = y^3 - y$

   (e) $j(x) = x^6 - x^2$

   (f) $k(x) = \sqrt{x^4 + x^2}$

   (g) $l(m) = (m+1)^2$

   (h) $a(x) = |x|$

   (i) $b(y) = |y-1|$