4.3 Logarithmic Functions (Definition of Logarithm)

- A logarithm base $b$ of a positive number $x$ satisfies the following definition.
  For $x > 0$, $b > 0$, $b \neq 1$, $y = \log_b(x)$ is equivalent to $b^y = x$. where,
- We read $\log_b(x)$ as, the logarithm with base $b$ of $x$ or the log base $b$ of $x$.”
- The logarithm $y$ is the exponent to which $b$ must be raised to get $x$.
- Also, since the logarithmic and exponential functions switch the $x$ and $y$ values, the domain and range of the exponential function are interchanged for the logarithmic function. Therefore, the domain of the logarithm function with base $b$ is $(0, \infty)$. the range of the logarithm function with base $b > 0$ is $(-\infty, \infty)$.

\[
\log_b(x) = y
\]

\[
b^y = x
\]

- A few graphs:

- The difference between graph of log functions in base $b > 1$ and $0 < b < 1$ is illustrated in the above graphs.
- The logarithm with natural base $e$ is denoted by $\ln$.
- The logarithm base 10 is denoted by $\log$, omitting the base.
1. Evaluate.

(a) $\log_3(3) = $  
(b) $\log_3(81) = $  
(c) $\log_9(81) = $  
(d) $\log_9(3) = $  
(e) $\log_9\left(\frac{1}{3}\right) = $  
(f) $\log_4(8) = $  
(g) $\log_2(1024) = $  
(h) $\log_2(.5) = $  
(i) $\log_4(\sqrt{2}) = $  
(j) $\log_2(\sqrt{2}) = $  
(k) $\log(10,000) = $  
(l) $\log(0.1) = $ 

2. Solve for $x$.

(a) $\log_3(x) = 2.$  
(b) $\log(x) = 5$  
(c) $x = \ln(e^2)$  
(d) $x = \ln(\sqrt{e})$ 

3. The intensity levels $I$ of two earthquakes measured on a seismograph can be compared by the formula $\log\left(\frac{I_1}{I_2}\right) = M_1 - M_2$ where $M_1$ and $M_2$ are the magnitudes given by the Richter Scale.

(a) How many times more intense is an earthquake of 7.1 than an earthquake of 6 Richter?
(b) If an earthquake is 20 times as intense as another, what is the difference of their Richter scale magnitude?