4.4: Graph of Logarithmic Function

- Graph of a logarithmic function $f(x) = \log_b(x)$ is reflection of its inverse $y = b^x$ over $y = x$.

- The logarithmic function only has a vertical asymptote and one x-intercept.

- Function $f(x) = a \log_b(cx - d) + f$ is obtained from $g(x) = \log_b(x)$ by a horizontal shift of $|d|$ units, possibly a reflection over y-axis if $c < 0$, a horizontal shrinking/stretching of ratio $|c|$, a possible reflection over x-axis if $a < 0$, a shrinking/stretching of ratio $|a|$ and a vertical shift $f$.

- Position of vertical asymptote for $f(x) = a \log_b(cx - d) + f$ is $x = \frac{d}{c}$.

- Position of vertical asymptotes for any log function can be found by setting inside of the log equal to zero.

- Position of x-intercept for any log function can be found by setting the function equal to zero. In case of a parent function for example, $f(x) = \log(g(x))$, set the inside function equal to one. Further finding the x-intercept is discussed in Section 4.6.

- Domain of log function can be found by setting inside of the log strictly bigger than zero and solving the resulting inequality.
1. Consider the function \( f(x) = \ln(2x^2 - 3x + 1) \).
   
   (a) Find its asymptote/s.
   (b) Find its domain.
   (c) Find the x-intercept/s.

**Solution.**

(a) \( 2x^2 - 3x + 1 = 0 \) \( \Rightarrow \) \( x = \frac{1}{2} \) and \( x = 1 \) (Two vertical asymptotes.)

(b) \( 2x^2 - 3x + 1 > 0 \) \( \Rightarrow \) \( x > 1 \) or \( x < 1/2 \). That is \( (-\infty, 1/2) \cup (1, \infty) \)

(c) \( \ln(2x^2 - 3x + 1) = 0 \) \( \Rightarrow \) \( 2x^2 - 3x + 1 = 1 \) \( \Rightarrow \) \( 2x^2 - 3x = 0 \) \( \Rightarrow \) \( x = 0 \) and \( x = 3/2 \)

2. Consider the function \( f(x) = 2 \ln(x + 3) + 1 \).
   
   (a) Find the function’s x-intercept.
   (b) Find its asymptotes.
   (c) Graph the function.
   (d) Find its domain.

**Solution.**

(a) \( 0 = 2 \ln(x + 3) + 1 \) \( \Rightarrow \) \( x + 3 = e^{-1/2} \) \( \Rightarrow \) \( x + 3 = \frac{1}{\sqrt{e}} \) \( \Rightarrow \) \( x = -3 + \frac{1}{\sqrt{e}} \) (x-intercept)

(b) \( x + 3 = 0 \) \( \Rightarrow \) \( x = -3 \) (Vertical asymptote)
3. Let $f(x) = \log_5(x - 1) + 7$. Then $f^{-1}(x) =$

(a) $ln(x - 7) + 1$
(b) $5x - 2$
(c) $5^{x-7} + 1$
(d) $\frac{x - 7}{5} + 1$

**Solution:**

Replace the function with a variable $y = \log_5(x - 1) + 7$

Solve for $x$: $y - 7 = \log_5(x - 1)$

$\underline{5^{y-7}} = x - 1$ $\Rightarrow$ $x = 1 + 5^{y-7}$

Switch variables: $y = 1 + 5^{x-7}$

$f^{-1}(x) = 1 + 5^{x-7}$