4.4: Graph of Logarithmic Function

- Graph of a logarithmic function $f(x) = \log_b(x)$ is reflection of its inverse $y = b^x$ over $y = x$.
- The logarithmic function only has a vertical asymptote and one x-intercept.
- Function $f(x) = a \log_b(cx - d) + f$ is obtained from $g(x) = \log_b(x)$ by a horizontal shift of $|d|$ units, possibly a reflection over y-axis if $c < 0$, a horizontal shrinking/stretching of ratio $|c|$, a possible reflection over x-axis if $a < 0$, a shrinking/stretching of ratio $|a|$ and a vertical shift $f$.
- Position of vertical asymptote for $f(x) = a \log_b(cx - d) + f$ is $x = \frac{d}{c}$.
- Position of vertical asymptotes for any log function can be found by setting inside of the log equal to zero.
- Position of x-intercept for any log function can be found by setting the function equal to zero. In case of a parent function for example, $f(x) = \log(g(x))$, set the inside function equal to one. Further finding the x-intercept is discussed in Section 4.6.
- Domain of log function can be found by setting inside of the log strictly bigger than zero and solving the resulting inequality.
1. Consider the function \( f(x) = \ln(2x^2 - 3x + 1) \).
   
   (a) Find its asymptote/s.
   (b) Find its domain.
   (c) Find the x-intercept/s.

2. Consider the function \( f(x) = 2 \ln(x + 3) + 1 \).
   
   (a) Find the function’s x-intercept.
   (b) Find its asymptotes.
   (c) Graph the function.
   (d) Find its domain.

3. Let \( f(x) = \log_5(x - 1) + 7 \). Then \( f^{-1}(x) = \)
   
   (a) \( \ln(x - 7) + 1 \)
   (b) \( 5x - 2 \)
   (c) \( 5^{x-7} + 1 \)
   (d) \( \frac{x - 7}{5} + 1 \)