5.3: The Other Trigonometric Functions

If \((x, y)\) is the terminal point for an angle \(t\).

- **Tangent of angle \(t\)**: \(\tan(t) = \frac{y}{x}\) for \(x \neq 0\). Also \(\tan(t) = \frac{\sin(t)}{\cos(t)}\) when \(\cos(t) \neq 0\).

- **Cotangent of angle \(t\)**: \(\cot(t) = \frac{x}{y}\) for \(y \neq 0\). Also \(\cot(t) = \frac{\cos(t)}{\sin(t)}\) when \(\sin(t) \neq 0\).

- **Secant of angle \(t\)**: \(\sec(t) = \frac{1}{x}\) for \(x \neq 0\). Also \(\sec(t) = \frac{1}{\cos(t)}\) when \(\cos(t) \neq 0\).

- **Cosecant \(t\)**: \(\csc(t) = \frac{1}{y}\) for \(y \neq 0\). Also \(\csc(t) = \frac{1}{\sin(t)}\).

- Another identity for **Tangent**: \(\tan(t) = \frac{1}{\cot(t)}\) when \(\cot(t) \neq 0\).

- Another identity for **Cotangent**: \(\cot(t) = \frac{1}{\tan(t)}\) when \(\tan(t) \neq 0\).

A function is **periodic** if there is a positive number \(p\) such that \(f(t) = f(t + p)\). That is the function repeats itself after time \(p\) has passed. We call \(p\) the period if it is the **smallest** such number.

- **Periodic properties of Sine and Cosine**:
  The function \(\sin(t)\) and \(\cos(t)\) have period \(2\pi\) that is \(\sin(t) = \sin(t + 2\pi)\) and \(\cos t = \cos(t + 2\pi)\) for all \(t\).
  We observed that the value of sine and cosine on the circle repeats itself after \(2n\pi\).

- **Pythagorean identities**:
  \[
  \sin^2(t) + \cos^2(t) = 1 \\
  \sec^2(t) = \tan^2(t) + 1 \\
  \csc^2(t) = 1 + \cot^2(t)
  \]

- **Even and Odd**:
  \[
  \text{Odd:} \quad \sin(-t) = -\sin(t) \quad \text{Even:} \quad \cos(-t) = \cos(t) \\
  \tan(-t) = -\tan(t) \quad \sec(-t) = \sec(t) \\
  \cot(-t) = -\cot(t) \quad \csc(-t) = -\csc(t)
  \]
• Finding Trigonometric Functions of \( \theta \)

• Find the reference angle, \( t \), for \( \theta \).
• Use the table of sine/cosine values for \( 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2} \) to find the absolute value of the sine and cosine.
• Subtract/add enough \( 2\pi \) to the angle to find the coterminal angle and its quadrant.
• Find the sign of the sine and cosine in the correct quadrant.
• Use the formula for \( \tan, \cot, \sec \) and \( \csc \) to find the other values.
• That is, \( \sin(\theta) = \pm(\sin(t)) \) and \( \cos(\theta) = \pm(\cos(t)) \)
1. Complete the table for all trig functions.

<table>
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<th>$t$</th>
<th>$\sin(t)$</th>
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<th>$\tan(t)$</th>
<th>$\cot(t)$</th>
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2. **Predator Prey Model**: In any model of population with predator and prey while the population of prey increases the population of predator increases. Then the increase in population of predator causes the population of the prey after a while causes the population of prey to decrease. And the decrease in population of prey causes the decrease in population of predator. This is a cycle that repeats itself and can be modeled by a simple periodic functions such as sine and cosine.

Let $N(t) = 1200 \sin(3t) + 2500$ be the population of prey over time. Find the maximum population and length of time between successive periods of maximum population.

3. The equation $P(t) = 20 \sin(2\pi t) + 100$ models the blood pressure, $P$, where $t$ represents time in seconds. (a) Find the blood pressure after 15 seconds. (b) What are the maximum and minimum blood pressures?