6.1: Graphs of Sine and Cosine

Using the points on the unit circle, graph the sine and the cosine of \( x \). For example, the points in the first quadrants will be as following.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( \cos(t) )</th>
<th>point</th>
<th>( t )</th>
<th>( \sin(t) )</th>
<th>point</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>((0,1))</td>
<td>0</td>
<td>0</td>
<td>((0,0))</td>
</tr>
<tr>
<td>( \pi )</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>((\pi \frac{\sqrt{3}}{6}, \frac{1}{2}))</td>
<td>( \pi )</td>
<td>( \frac{1}{2} )</td>
<td>((\pi \frac{1}{6}, \frac{1}{2}))</td>
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<td>( \frac{\pi}{6} )</td>
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<td>( \frac{\pi}{6} )</td>
<td>( \frac{1}{2} )</td>
<td>((\pi \frac{1}{6}, \frac{1}{2}))</td>
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<tr>
<td>( \frac{\pi}{4} )</td>
<td>( \frac{\sqrt{2}}{2} )</td>
<td>((\pi \frac{\sqrt{2}}{4}, \frac{1}{2}))</td>
<td>( \frac{\pi}{4} )</td>
<td>( \frac{\sqrt{2}}{2} )</td>
<td>((\pi \frac{\sqrt{2}}{4}, \frac{1}{2}))</td>
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<td>( \frac{\sqrt{3}}{2} )</td>
<td>((\pi \frac{\sqrt{3}}{3}, \frac{1}{2}))</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>0</td>
<td>((\pi \frac{1}{2}, 0))</td>
<td>( \frac{\pi}{2} )</td>
<td>1</td>
<td>((\pi \frac{1}{2}, 1))</td>
</tr>
</tbody>
</table>

Period = 2\( \pi \)

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**Graph of \( y = \cos(x) \)**

**Graph of \( y = \sin(x) \)**

Period = 2\( \pi \)
• Comparing sine functions;

- A sinusoidal function any function that can be expressed in the form \( f(x) = A \sin(Bx - C) + D \) or \( f(x) = A \cos(Bx - C) + D \).

- **Midline:** The horizontal line \( y = D \), where \( D \) appears in the general form of a sinusoidal function. (It is called midline because \( D \) is the average y-value.)

- **Amplitude:** The vertical height of a function form Midline; the constant \( A \) appearing in the definition of a sinusoidal function.

- **A periodic function:** A function \( f(x) \) that satisfies \( f(x + P) = f(x) \) for a specific constant \( P \) and any value of \( x \). \((P \) is the smallest positive value that satisfies such equation and is called the Period\.) The formula \( P = \frac{2\pi}{B} \) gives the period.

- **Phase shift** The horizontal displacement of the basic sine or cosine function; the constant \( \frac{C}{B} \) for \(-2\pi < C < 2\pi\).

**Transformations:**

- The above picture can be explained using transformations as well but the formulas that came easier.

- **How to graph:** Find the local max and min points, amplitude, period, phase shift and vertical transformation and graph.
1. Sketch two periods of the graph of \( y = \frac{1}{3} \sin(2x - \frac{\pi}{2}) \), labeling the maximum and minimum height, the \( x \)-intercepts and two more points on one period. List the amplitude, period and phase shift of \( f(x) \).

**Solution:**

- **The Amplitude is** \( A = \frac{1}{3} \)
- **Phase shift:** \( 2x - \frac{\pi}{2} = 0 \) gives \( x = \frac{\pi}{4} \) (Also an \( x \)-intercept)
- **Period:** \( P = \frac{2\pi}{2} = \pi \)
- **The end of the first period is:** \( 2x - \frac{\pi}{2} = 2\pi \) so \( x = \frac{5\pi}{4} \) (Also an \( x \)-intercept)
- **The \( x \)-intercept in the middle comes from** \( 2x - \frac{\pi}{2} = \pi \) which gives \( x = \frac{3\pi}{4} \)
- **One maximum at** \( 2x - \frac{\pi}{2} = \pi/2 \) which is \( x = \frac{\pi}{4} \) which corresponds to point \( \left( \frac{\pi}{2}, \frac{1}{3} \right) \)
- **One minimum at** \( 2x - \frac{\pi}{2} = 3\pi/2 \) which is \( x = \pi \) and the corresponding point is \( \left( \pi, -\frac{1}{3} \right) \)
2. A weight is attached to a spring that is then hung from a board, as shown in Figure. As the spring oscillates up and down, the position \( y \) of the weight relative to the board ranges from \(-2\) in. (at time \( t = 0 \) second) to \(-6\) in. (at time \( t = 2\pi \) second) below the board. Assume the position \( y \) is given as a sinusoidal function of \( t \). Sketch a graph of the function, and then find a cosine function that gives the position \( y \) in terms of \( t \). Motion of this spring mass system is a simple harmonic motion.

**Solution:**
Amplitude is \( \frac{-2 - (-6)}{2} = 2 \) and The period is \( 4\pi \) \( \Rightarrow \) \( B = \frac{2\pi}{4\pi} = 0.5 \), phase shift is 0 with a sine function and the average displacement is \( \frac{-2 - 6}{2} = -4 \) which gives the vertical shift \( D = -4 \)
So \( f(t) = 2\cos(0.5t) - 4 \) because the initial value is a maximum.

3. Find a function that models the simple harmonic motion having Period 4 and amplitude 10. Assume that the displacement is zero at time \( t = 0 \).

**Solution:**
\[ P = \frac{2\pi}{B} \Rightarrow B = \frac{2\pi}{P} = \frac{\pi}{2} \]
\[ A = 10 \]
Initial displacement is zero. We should use sine function with phase shift equal to zero.
So the function is \( f(t) = 10\sin\left(\frac{\pi t}{2}\right) \) and \( f(t) = -10\sin\left(\frac{\pi t}{2}\right) \) is also acceptable.
4. A Ferris wheel’s radius is 25 meters and boarded from a platform that is 1 meter above the ground. The six oclock position on the Ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 10 minutes. The function h(t) gives a person’s height in meters above the ground t minutes after the wheel begins to turn.

(a) Find the amplitude, midline, and period of h(t).
(b) Find a formula for the height function h(t).
(c) How high off the ground is a person after 5 minutes?

\[ h(t) = 25 \sin \left( \frac{\pi}{5} t - \frac{\pi}{2} \right) + 26 \]

**Solution:**

(a) Amplitude is \( A = 25 \), Midline is \( y = 26 \) and period is 10 minutes. So \( B = \frac{2\pi}{10} = \frac{\pi}{5} \).

(b) \( h(t) = 25 \sin \left( \frac{\pi}{5} t - \frac{\pi}{2} \right) + 26 \)

(c) \( h(5) = 25 \sin \left( \pi - \frac{\pi}{2} \right) + 26 = 51 \) meters above the ground.

5. The period of \( f(x) = 2 \cos(4x + \pi/6) \) is

(a) \( 2\pi \)
(b) \( \pi/2 \)
(c) \( \pi/2 \)
(d) \( 4\pi \)

**Solution:** \( \frac{2\pi}{4} = \pi/2 \)
6. Graph

\[ f(x) = \begin{cases} 
  x & x < -2 \\
  \sin(x) & -2 \leq x \leq 0 \\
  -x^2 & 0 < x < 2 \\
  \cos(x) & x \geq 2 
\end{cases} \]

**Solution:**

\[ f(x) = \begin{cases} 
  x & x < -2 \\
  \sin(x) & -2 \leq x \leq 0 \\
  -x^2 & 0 < x < 2 \\
  \cos(x) & x \geq 2 
\end{cases} \]
A few Videos

1. **Graph of Sine and Cosine Functions 1:**
   https://mediahub.ku.edu/media/MATH+-+Graph+of+Sine+and+Cosine+Functions+1.m4v/1_zqn7xygk

2. **Graph of Sine and Cosine Functions 2:**
   https://mediahub.ku.edu/media/MATH++-Graph+of+Sine+and+Cosine+Functions+2.m4v/1_3i8ik9rt