Final Exam Review Questions

1. Compute the following integrals.

(a) \( \int \frac{3x^3 - x^2 - 4}{(x - 2)^2(x^2 + 4)} \, dx \).

(b) \( \int \sin^4(x) \cos^3(x) \, dx \).

(c) \( \int x^4(\ln(x))^2 \, dx \).

(d) \( \int \frac{1}{x\sqrt{x^2 + 1}} \, dx \).
2. Determine the values of $p$ for which the following integrals converge.

(a) $\int_0^1 \frac{1}{x^p} \, dx$.

(b) $\int_1^\infty \frac{1}{x^p} \, dx$.

3. Determine if each of the following integrals is convergent or divergent.

(a) $\int_0^\infty \frac{1}{\sqrt{x}(x+2)^2} \, dx$.

(b) $\int_{-\infty}^\infty \frac{1}{1+x^2} \, dx$.

(c) $\int_1^\infty \frac{1}{(x-2)^2} \, dx$. 
4. Let \( f(x) = \sin(x) \) and \( g(x) = \sin(2x) \). Consider the region \( R \) bounded by \( f(x) \) and \( g(x) \) on the interval \([0, \pi/6]\).

(a) Sketch the region \( R \).

(b) Find the area of the region \( R \).

(c) Find the volume of the solid generated by revolving the region \( R \) about the \( x \)-axis.

(d) Find the volume of the solid generated by revolving the region \( R \) about the \( y \)-axis.

(e) Set up integrals to compute the length of the boundary of the region \( R \).
5. Find all sixth roots of \(1 + i\).

6. Find all solutions to the equation \(z^5 = 32i\).
7. Determine if the following series are absolutely convergent, conditionally convergent, or divergent.

(a) \[ \sum_{n=1}^{\infty} \frac{n^4 + 2n^2 - n + 1}{n^2 + 2n + 4} \]

(b) \[ \sum_{n=2}^{\infty} \frac{1}{n (\ln(n))^{1/2}} \]

(c) \[ \sum_{n=1}^{\infty} \frac{n^{20}}{(1.1)^n} \]

(d) \[ \sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln(n))^2} \]

(e) \[ \sum_{n=1}^{\infty} \frac{(-1)^n n^2}{(n + 1)^2} \]
8. Suppose a power series \( \sum a_n x^n \) converges when \( x = -2 \) and diverges when \( x = 6 \).

(a) Does the power series converge at \( x = 1 \)?

(b) Does the power series converge at \( x = -7 \)?

(c) Does the power series converge at \( x = -6 \)?

(d) What can you say about the radius of convergence of the power series?

9. Find the Taylor series for the following functions centered at the given point. Also give the radius of convergence.

(a) \( f(x) = xe^x; \ a = 0 \).

(b) \( f(x) = xe^x; \ a = 4 \).

(c) \( f(x) = \cos(x^2); \ a = 0 \).

(d) \( f(x) = \frac{1}{x}; \ a = 3 \).

(e) \( f(x) = \frac{1}{x^2}; \ a = 3 \).
10. Consider the curve given by the polar equation $r = \theta^2 - \theta$.

(a) Sketch the curve for $0 \leq \theta \leq 3\pi$.

(b) Find the derivative $\frac{dy}{dx}$ as a function of $\theta$.

(c) Find the equation of the line tangent to the curve at the point $(r, \theta) = (4\pi^2 - 2\pi, 2\pi)$
11. A random variable $X$ has distribution $f(x) = \begin{cases} \frac{k}{x^2} & x \geq 1 \\ 0 & x < 1 \end{cases}$

(a) Find the value of $k$ so that $f$ is a distribution.

(b) Find the mean.

(c) Find the probability $P(4 \leq X \leq 10)$. 