
- **FAQ.** What is the role of letters in mathematics?

- **Answer.** Letters represent numbers.

  In other words, a letter is to be substituted by a number.

Today, I only use the letter \( n \).

**Example 1.** Substitute \( n = 5 \) in \( n + 1 \).

**Solution.** \( 5 + 1 = 6 \).

**Example 2.** Substitute \( n = 3 \) in \( n + 2 \).

**Solution.** \( 3 + 2 = 5 \).

**Example 3.** Substitute \( n = 11 \) in \( n + 3 \).

**Solution.** \( 11 + 3 = 14 \).
Example 4. Substitute $n = 20$ in $n + 4$.

Solution. $20 + 4 = 24$.

Example 5. Substitute $n = 1$ in $\frac{1}{2}n(n + 1)$.

Solution. $\frac{1}{2} \cdot 1 \cdot 2 = 1$.

Example 6. Substitute $n = 4$ in $\frac{1}{2}n(n + 1)$.

Solution. $\frac{1}{2} \cdot 4 \cdot 5 = 10$.

Example 7. Substitute $n = 6$ in $\frac{1}{6}n(n + 1)(n + 2)$.

Solution. $\frac{1}{6} \cdot 6 \cdot 7 \cdot 8 = 56$.

Example 8. Substitute $n = 9$ in $\frac{1}{24}n(n + 1)(n + 2)(n + 3)$.

Solution. $\frac{1}{24} \cdot 9 \cdot 10 \cdot 11 \cdot 12 = 495$. 

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Get used to expressions like

(a) \[1 + 2 + 3 + 4 + 5 + \cdots + n,\]

(b) \[1 + 3 + 6 + 10 + 15 + \cdots + \frac{1}{2}n(n + 1),\]

(c) \[1 + 4 + 10 + 15 + 21 + \cdots + \frac{1}{6}n(n + 1)(n + 2).\]

The meaning of (a) is self-evident.

(b) is as follows:

\[
1 + 3 + 6 + 10 + 15 + \cdots + \frac{1}{2}n(n + 1).
\]

Arrows point the outcomes of substituting the indicated numbers in \(\frac{1}{2}n(n + 1)\).

Similarly, (c) is as follows:

\[
1 + 4 + 10 + 20 + 35 + \cdots + \frac{1}{6}n(n + 1)(n + 2).
\]

Once again, arrows point the outcomes of substituting the indicated numbers in \(\frac{1}{6}n(n + 1)(n + 2)\).
Example 9. Substitute $n = 8$ in $1 + 2 + 3 + 4 + 5 + \cdots + n$.

First spell out the outcome, and then calculate it.

Solution. $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$.

Example 10. Substitute $n = 6$ in $1 + 3 + 6 + 10 + \cdots + \frac{1}{2}n(n+1)$.

First spell out the outcome, and then calculate it.

Solution. $1 + 3 + 6 + 10 + 15 + 21 = 56$.

- Shift.

Among important operations in mathematics is the ‘shift’ of the letter $n$. Shift means you

$\text{substitute } n \text{ by } n + 1$.

So, every single $n$ spotted in the given formation is getting replaced with $n + 1$.

We denote the shift by $n \mapsto n + 1$.

Example 11. Shift $n \mapsto n + 1$ in $n + 3$.

Solution. $(n+1) + 3 = n + 4$. 

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Example 12. Shift \( n \mapsto n + 1 \) in \( n + 10 \).

Solution. \((n + 1) + 10 = n + 11\).

Example 13. Shift \( n \mapsto n + 1 \) in \( \frac{1}{2}n(n + 1) \).

Solution. \( \frac{1}{2}(n + 1)(n + 2) \).

Example 14. Shift \( n \mapsto n + 1 \) in \( \frac{1}{6}n(n + 1)(n + 2) \).

Solution. \( \frac{1}{6}(n + 1)(n + 2)(n + 3) \).

Example 15. Shift \( n \mapsto n + 1 \) in

\[
1 + 3 + 6 + 10 + 15 + \cdots + \frac{1}{2}n(n + 1)
\]

Solution.

\[
1 + 3 + 6 + 10 + 15 + \cdots + \frac{1}{2}(n + 1)(n + 2).
\]