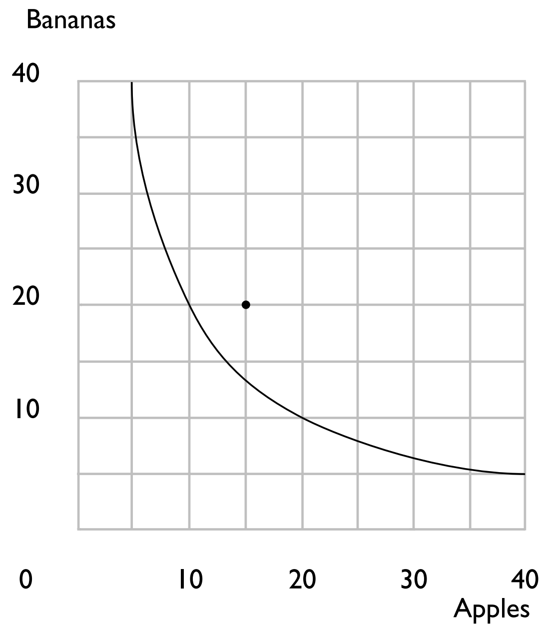
$u(x_1, x_2)$	$MU_1(x_1, x_2)$	$MU_2(x_1, x_2)$	$MRS(x_1, x_2)$
$2x_1 + 3x_2$	2	3	$-2/3$
$4x_1 + 6x_2$	4	6	$-2/3$
$ax_1 + bx_2$	a	b	$-a/b$
$2\sqrt{x_1} + x_2$	$\frac{1}{\sqrt{x_1}}$	1	$-\frac{1}{\sqrt{x_1}}$
$\ln x_1 + x_2$	$1/x_1$	1	$-1/x_1$
$v(x_1) + x_2$	$v'(x_1)$	1	$-v'(x_1)$
$x_1 x_2$	x_2	x_1	$-x_2/x_1$
$x_1^a x_2^b$	$ax_1^{a-1}x_2^b$	$bx_1^a x_2^{b-1}$	$-\frac{ax_2}{bx_1}$
$(x_1 + 2)(x_2 + 1)$	$x_2 + 1$	$x_1 + 2$	$-\left(\frac{x_2+1}{x_1+2}\right)$
$(x_1 + a)(x_2 + b)$	$x_2 + b$	$x_1 + a$	$-\left(\frac{x_2+b}{x_1+a}\right)$
$x_1^a + x_2^a$	ax_1^{a-1}	ax_2^{a-1}	$-\left(\frac{x_1}{x_2}\right)^{a-1}$

4.1 (0) Remember Charlie from Chapter 3? Charlie consumes apples and bananas. We had a look at two of his indifference curves. In this problem we give you enough information so you can find *all* of Charlie's indifference curves. We do this by telling you that Charlie's utility function happens to be $U(x_A, x_B) = x_A x_B$.

(a) Charlie has 40 apples and 5 bananas. Charlie's utility for the bundle $(40, 5)$ is $U(40, 5) =$ **200.** The indifference curve through $(40, 5)$ includes all commodity bundles (x_A, x_B) such that $x_A x_B =$ **200.** So the indifference curve through $(40, 5)$ has the equation $x_B = \frac{200}{x_A}$**.** On the graph below, draw the indifference curve showing all of the bundles that Charlie likes exactly as well as the bundle $(40, 5)$.

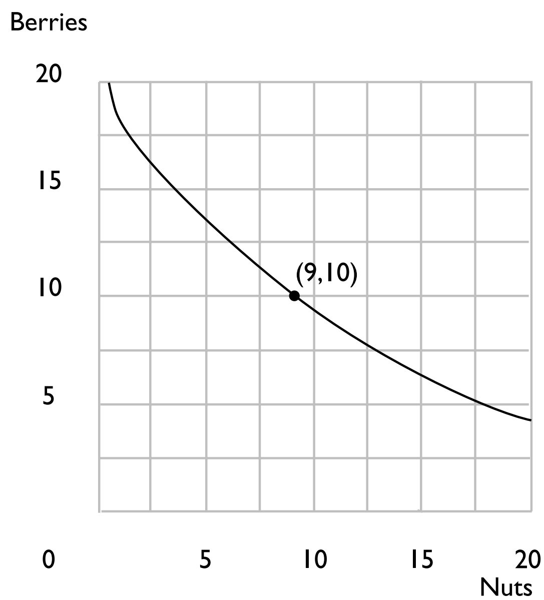


(b) Donna offers to give Charlie 15 bananas if he will give her 25 apples. Would Charlie have a bundle that he likes better than $(40, 5)$ if he makes this trade? **Yes.** What is the largest number of apples that Donna could demand from Charlie in return for 15 bananas if she expects him to be willing to trade or at least indifferent about trading? **30.** (Hint: If Donna gives Charlie 15 bananas, he will have a total of 20 bananas. If he has 20 bananas, how many apples does he need in order to be as well-off as he would be without trade?)

4.2 (0) Ambrose, whom you met in the last chapter, continues to thrive on nuts and berries. You saw two of his indifference curves. One indifference curve had the equation $x_2 = 20 - 4\sqrt{x_1}$, and another indifference curve had the equation $x_2 = 24 - 4\sqrt{x_1}$, where x_1 is his consumption of nuts and x_2 is his consumption of berries. Now it can be told that Ambrose has quasilinear utility. In fact, his preferences can be represented by the utility function $U(x_1, x_2) = 4\sqrt{x_1} + x_2$.

(a) Ambrose originally consumed 9 units of nuts and 10 units of berries. His consumption of nuts is reduced to 4 units, but he is given enough berries so that he is just as well-off as he was before. After the change, how many units of berries does Ambrose consume? 14.

(b) On the graph below, indicate Ambrose's original consumption and sketch an indifference curve passing through this point. As you can verify, Ambrose is indifferent between the bundle (9,10) and the bundle (25,2). If you doubled the amount of each good in each bundle, you would have bundles (18,20) and (50,4). Are these two bundles on the same indifference curve? No. (Hint: How do you check whether two bundles are indifferent when you know the utility function?)



(c) What is Ambrose's marginal rate of substitution, $MRS(x_1, x_2)$, when he is consuming the bundle $(9, 10)$? (Give a numerical answer.) $-2/3$. What is Ambrose's marginal rate of substitution when he is consuming the bundle $(9, 20)$? $-2/3$.

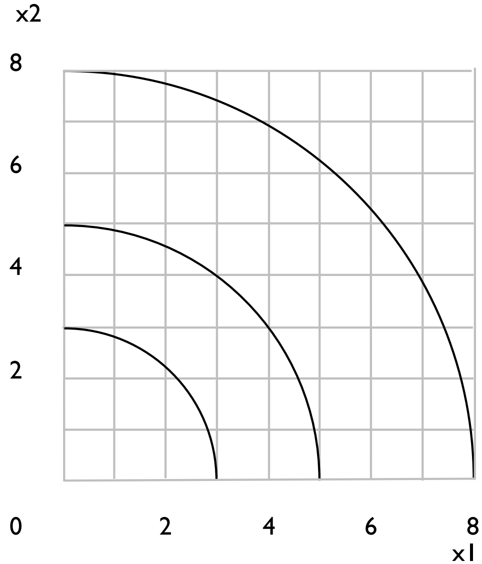
(d) We can write a general expression for Ambrose's marginal rate of substitution when he is consuming commodity bundle (x_1, x_2). This is $MRS(x_1, x_2) =$ $-2/\sqrt{x_1}$. Although we always write $MRS(x_1, x_2)$ as a function of the two variables, x_1 and x_2, we see that Ambrose's utility function has the special property that his marginal rate of substitution does not change when the variable x_2 changes.

4.5 (0) As you may recall, Nancy Lerner is taking Professor Stern's economics course. She will take two examinations in the course, and her score for the course is the minimum of the scores that she gets on the two exams. Nancy wants to get the highest possible score for the course.

(a) Write a utility function that represents Nancy's preferences over alternative combinations of test scores x_1 and x_2 on tests 1 and 2 respectively. $U(x_1, x_2) =$ $\min\{x_1, x_2\}$, or any monotonic transformation.

4.11 (0) Willy Wheeler's preferences over bundles that contain non-negative amounts of x_1 and x_2 are represented by the utility function $U(x_1, x_2) = x_1^2 + x_2^2$.

(a) Draw a few of his indifference curves. What kind of geometric figure are they? `Quarter circles centered at the origin.` Does Willy have convex preferences? `No.`



Calculus **4.12 (0)** Joe Bob has a utility function given by $u(x_1, x_2) = x_1^2 + 2x_1x_2 + x_2^2$.

(a) Compute Joe Bob's marginal rate of substitution: $MRS(x_1, x_2) =$

-1.

(b) Joe Bob's straight cousin, Al, has a utility function $v(x_1, x_2) = x_2 + x_1$. Compute Al's marginal rate of substitution. $MRS(x_1, x_2) =$ -1.

(c) Do $u(x_1, x_2)$ and $v(x_1, x_2)$ represent the same preferences? Yes. Can you show that Joe Bob's utility function is a monotonic transformation of Al's? (Hint: Some have said that Joe Bob is square.) Notice that $u(x_1, x_2) = [v(x_1, x_2)]^2$.