Dirty Secrets of the TI83 and TI84

(some of them—not all of them)

1 The small-differences-incorrectly-set-to-zero "feature"

On a TI83 or TI84, the result of evaluating the expression

$$1 + 1\text{E}^{-}13 - 1$$

is 0. The similar expression

$$1 + 1\text{E}^{-}13 - .9 - .1$$

evaluates to 1E⁻13 which is exactly correct. Note that *there is no rounding error in either expression.*

A few graphing calculators, including TI83's and IT84's have the "feature" that small differences are incorrectly set to zero. On these calculators, whenever a nonzero difference $a - b$ has small enough magnitude, it is set to zero. To be precise, TI83's and TI84's have the following small-differences-incorrectly-set-to-zero "feature".

> If a and b are floating point numbers that differ by less than one unit in the second least significant digit of $\max(|a|, |b|)$, (i.e., one unit the thirteenth, base 10, significant digit), then $a - b$ is set to zero.

Of course, the rule applies equally to sums of numbers of opposite sign.

TI83's and IT84's use 14-digit, base 10 arithmetic. The exact result of each arithmetic operation is a floating point number, so no rounding is necessary to change a non-floating point number to a floating point number. In the first expression, final subtraction is

$$1.0000000000001 \times 10^0 - 1.0000000000000 \times 10^0.$$

In this example, the second least significant digit's place is the 10^{-12}'s place. The two operands differ by 10^{-13} which is less than one unit in the second-least significant digit's place, i.e., less than 10^{-12}. The small-differences-incorrectly-set-to-zero "feature" sets the result incorrectly to zero. In the second expression, the final subtraction is

$$1.0000000000010 \times 10^{-1} - 1.0000000000000 \times 10^{-1}$$

In this example, the second least significant digit's place is the the 10^{-13}'s place. The two operands differ by only one unit the second least significant digit's place, but that is enough to prevent the small-differences-incorrectly-set-to-zero "feature" from setting the difference to zero.

On a TI86, both $1 + 1\text{E}^{-}13 - 1$ and $1 + 1\text{E}^{-}13 - .9 - .1$ give the correct result $1\text{E}^{-}13$; the TI86 does not have the small-differences-incorrectly-set-to-zero "feature."

Baring exponent underflow, the floating point difference of two nearly equal floating point numbers is a floating point number, so the difference is calculated *without rounding error.* Consequently, every time the small-differences-incorrectly-set-to-zero "feature" sets a difference to zero, it inserts an error into an operation that floating point arithmetic would otherwise get exactly, mathematically correct!

2 The different-numbers-are-equal "feature"

According to the TI83 Plus Graphing Calculator GuideBook,

> ...the expression $valueA = valueB$ evaluates to `1` (true) if $valueA$ is equal to $valueB$. The expression $valueA = valueB$ evaluates to `0` (false) if $valueA \neq valueB$ where $valueA$ and $valueB$ may be real or complex numbers, expressions, lists or matrices.

This is just what one would expect. Ordinarily, rounding errors do not affect comparisons. Nevertheless, on a TI83 or TI84, expression

$$1 + 4\text{E}^{-}10 = 1$$

evaluates to 1, i.e., true! The similar expression

$$2 + 8\text{E}^{-}10 = 2$$

evaluates correctly to 0, i.e. false. Note that *no rounding error occurs when evaluating these expressions.*

The TI83's and TI84's have the following different-numbers-are-equal "feature".

> If a and b are floating point numbers that differ by less than five units in the eleventh significant digit of $\max(|a|, |b|)$, then $a = b$ evaluates to 1 (and $a < b$, $a > b$ both evaluate to 0).

In the expression, $1 + 4\text{E}^{-}10 = 1$, $a = 1 + 4\text{E}^{-}10$ and $b = 1$. The eleventh significant digit in both a and b is the 10^{-10}'s place. Five units in the eleventh significant digit is 5×10^{-10}. The difference $a - b$ is $4\text{E}^{-}10$ which is less than 5^{-10}, so the different-numbers-are-equal "feature" makes $1 + 4\text{E}^{-}10$ "equal" to 1.

In the expression, $2 + 8\text{E}^{-}10 = 2$, $a = 2 + 8\text{E}^{-}10$ and $b = 2$. As above, the eleventh significant digit in both a and b is the 10^{-10}'s place. Five units in the eleventh significant digit is 5×10^{-10}. The difference $a - b$ is $8\text{E}^{-}10$ which is greater than or equal to 5×10^{-10}, so the different-numbers-are-equal "feature" is inactive and $2 + 8\text{E}^{-}10$ remains "greater than" 2 as it should be.

3 Conclusion

Are the small-differences-incorrectly-set-to-zero feature and the different-numbers-are-equal feature undesirable? The answer is both yes and no!

I can only speculate why the manufacturers would choose to change correct results into incorrect results. (Texas Instruments is not the only calculator manufacturer that engages in such dubious practices.) Possibly the "features" were built-in so that simple expressions like $3 * (1/3) - 1$ evaluate to zero. However, the slightly more complicated expression $3(1/3) - .99 - .01$ does not evaluate to zero. The cost of having $3(1/3) - 1 = 0$ is that calculator programs are mysteriously difficult to debug and calculations are rarely as accurate as fourteen significant digit arithmetic can produce. Frequently, calculator programs behave as if there were only ten or eleven significant digits instead of fourteen. It is a high price to pay for an insignificant nicety.

On the-other-hand, these calculators are clearly designed to accompany elementary pre-calculus and calculus courses in mathematics. Traditionally, mathematics textbook examples and exercises are strongly biased toward convenient hand calculation. The intermediate quantities and answers are almost always small integers or ratios of small integers. The elementary mathematics student finds himself/herself learning looking-glass-land mathematics in which there is a Bayesian prior distribution that requires intermediate quantities and answers to be small integers or ratios of small integers with probability $1 - \varepsilon$. So, in the looking-glass land, .9999999999999 and $1\text{E}^{-}14$ are highly unlikely numbers: nearby quantities 1 and 0, respectively, are much more likely to be correct. The small-differences-incorrectly-set-to-zero feature and the different-numbers-are-equal feature are well suited to looking-glass-land mathematics.

Exercises:

1. Find a pair of numbers `A` and `B` for which the expressions $\texttt{A} > \texttt{B}$, $\texttt{A} < \texttt{B}$ and $\texttt{A} - \texttt{B} = 0$ are all false (0) on a TI83 or TI84.

2. (Difficult!) Find a way to test whether two numbers are equal on a TI83 or TI84. (HINT: Although $\texttt{A} - \texttt{B} = 0$ is much more accurate than $\texttt{A} = \texttt{B}$, the expression $\texttt{A} - \texttt{B}$ might trigger the small-differences-incorrectly-set-to-zero feature and yield an incorrect result.)

3. (Very difficult!) Find a way to correctly evaluate the difference $\texttt{A} - \texttt{B}$ in a TI83 or TI84 calculator.

4. TI83's and TI84's are not the only calculators with dubious "features". By experimenting, find a similar "feature" of some other brand of calculator or an Excel spreadsheet.