MATH 116: Final - Sample

You may use neither books nor notes. Work individually. Show the key intermediate steps to receive full credits.

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<tr>
<th>Problem</th>
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1. (20 points) Find the derivatives or partial derivatives of the following functions.

(a) \( f(x, y) = x^2 + y^2 + 4xy \). \( f_x =? \quad f_y =? \quad f_{xy} =? \)

(b) \( f(x) = \tan(x) \). \( f' =? \)

(c) \( f(x, y) = \sin(x^2y) \). \( f_x =? \quad f_y =? \)

(d) \( f(x) = \frac{1}{\sqrt{x}} \). \( f' =? \)

2. (20 points) Find the following integrals (indefinite, definite, and improper).

(a) \( \int_1^9 \left( \frac{1}{\sqrt{x}} + \sqrt{x} \right) \, dx \).

(b) \( \int \frac{\cos t}{\sqrt{\sin t}} \, dt \).

(c) \( \int_{\pi/6}^{\pi/2} \cos(2t) \, dt \). (Evaluate this integral by hands and leave the answer in exact form.)

(d) \( \int_1^\infty \frac{1}{x^2} \, dx \).
3. (15 points) Find the maximum and minimum values of function $f(x, y) = 4xy$ subject to constraint $x^2 + y^2 = 32$. (Both extreme values do exist.)

4. (15 points) What is the general solution of the differential equation $y' = \frac{y}{x}$? What is the particular solution that satisfies the initial condition $y(1) = 1$?
5. (15 points) Find the area between curves \( y = \cos(x) \) and \( y = \sin(x) \) from \( x = 0 \) to \( x = \pi/2 \).

6. (15 points) Find \( \int x^2 \cos(x) \, dx \) using the integration-by-parts technique.