A hydrodynamic simulation for the circulation and transport in coastal watersheds

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1. Introduction

The watersheds in the northern Gulf Coast between east Texas and west Florida have significant environmental importance. Some of these watersheds are the major water sources of the surrounding wetlands, thus are critical to the health of these wetlands. Northern Gulf Coast is also a heavily industrial area, and those watersheds often serve as water pathways for transporting the materials and products for the industry. Understanding the circulation and sedimentations will also contribute to the safety of navigation. Two typical watersheds have been chosen to study in this research, and they are Calcasieu Lake and Mobile Bay. They are both downstream of a heavily industrial city and port, Lake Charles and Mobile, respectively. The shapes of the two watersheds are similar and both have a ship channel connecting the city to the Gulf of Mexico. However, one of the major differences is their exit geometries. This causes differences in circulation and particulate transport patterns in these two watersheds.

Calcasieu Lake is a shallow water lake located in the middle of the Chenier Plain Wetlands on the northern coast of the Gulf of Mexico extended from Vamilian Parish of Louisiana to Chambers County of Texas. It has a surface area of 256 km² and an average depth of approximately 1.5 m. Calcasieu Lake is downstream of a heavily industrial area, the city of Lake Charles, and also the major water sources to two nearby National Wildlife Refuges (NWR) – Cameron Prairie NWR and Sabine NWR – that are covered mostly by coastal wetlands. Therefore, the pollution, nutrient and sediment transport in Calcasieu Lake is critical to the health of the wetlands. The health of the wetlands is not only necessary for the preservation and continued survival and growth of vegetative, aquatic and migratory species, but also for the related productive use and enjoyment of these marsh areas and recreation. Without reclamation and the erosion of the wetlands, eventually lead to navigation safety problems. These excessive sedimentations result in much shallower water depth that will affect the navigation safety problems. The results also show the effects of geometry on flow and flow-related transport.
sensitivity of coastal circulations (resulting from tidal currents and freshwater inputs, including stormwater runoff) can be evaluated. Possible additions of water quality components to the model, such as nutrient and sediment loading from river inputs, stormwater and wastewater treatment facilities, can greatly improve our understanding of sediment transport (with dredging implications), oxygen depletion (with fisheries implications), and possibly pathogen transport (with public health implications).

The current computational model is based on Spall [4], which is modified from Casulli’s [5,6] algorithm. The two-dimensional shallow water equations were solved using a semi-implicit algorithm with an alternating-direction-implicit (ADI) technique. A blended first- and second-order upwind approximation was employed for convection terms. Staggered grids have been used in these methods.

Both curvilinear and Cartesian coordinate systems were used in shallow-water simulations. The curvilinear coordinate system was used in several recent simulations [7–11]. Simulations of similar watersheds using the Cartesian coordinate system includes Gross et al. who conducted the Eulerian type of salinity transport simulation for South San Francisco Bay [12]. Liang and Borthwick applied an immersed-boundary type of method to treat curved or irregular boundaries in Cartesian grid shallow flow models [13]. None of these studies used Lagrangian particle tracers to investigate the particle transport.

Since the targeted simulation areas can be efficiently covered by a rectangular Cartesian domain, there are not many grid points wasted outside the areas, contrary to the Sloan river case in [4]. In addition, using the Cartesian approach is easier to implement grid-nesting techniques for subsequent research, enabling a more rigorous grid-resolution convergence check and computational scheme stability analysis. This is because the grid convergence is usually checked by reducing the grid size (and thus increasing the grid number). The grid size reduction can be achieved in a more controlled way for a structured grid than for an unstructured grid because of the orderly arrangement of the structured grid. Similarly, the computational stability condition that determines the size of the time step depends on the grid size, as shown later. A structured grid mesh, in this case a Cartesian grid mesh, gives a definitive criterion of the time step based on the definitive grid size at each grid. Therefore, in this paper, the Cartesian grid results are presented.

Figs. 1 and 2 are the water depth contours of Calcasieu Lake and Mobile Bay from the bathymetric data from NGLI [14] mapped to the computational grids. An earlier version of the Mobile Bay study [15] was based on an old version of the bathymetric data from NOAA published in 1998 [16]. The major differences between the two data sets are in the shapes of Dauphine Island at the entrance of Mobile Bay. The rectangular frame is the indication of the actual computational domain. The unit of the numbers on the color bar is meter. For Calcasieu Lake, the lake water and ocean water exchange through a narrow water passage in the south of the lake, named Calcasieu Pass. In Fig. 1b, the depth of Calcasieu Ship Channel is not included in the data. Therefore, a 12m-deep ship channel is manually added using software DigXY conforming with the actual geometry of the channel, and mapped into the computational grids, as shown in Fig. 1a. In Fig. 2, the main ship channel of the Mobile Bay (the dashed depth line) is resolved by the grids. A small island near the mid-bay area, Gaillard Island, is also included, while in [15] it was not due to the resolution of the NOAA data [16], although its effects on the overall circulation were minimal.

The simulation results include effects from Calcasieu River on the north of Calcasieu Lake, and the four major rivers around the Mobile Bay area: Mobile River, Tensaw River, Blakeley River and Dog River, as indicated in Fig. 2. Flow patterns and elevation contours are shown during flood and ebb tides. In Calcasieu Lake simulation, a set of ocean-surface elevation data from NOAA [17] was used as the outflow boundary conditions. For Mobile Bay simulation, a sinusoidal tide model has been used at the outflow due to the lack of actual measurement data. Velocity vectors and particle tracers are used to indicate the flow patterns and related transport of the circulations. To the authors’ knowledge, this study is the first that the flow and mass transports in these two areas have been analyzed and compared in such a detailed level.

Fig. 1. Water depth and computational domain of Calcasieu Lake (unit in meters). (a) With Calcasieu Ship Channel, (b) without Calcasieu Ship Channel.

2. Computational model

The current computational method is based on [4], which is modified from Casulli’s [5] algorithm. The two-dimensional shallow water equations to be solved have the form

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{\partial z}{\partial x} - \frac{\partial \zeta}{\partial x}, \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{\partial z}{\partial y} - \frac{\partial \zeta}{\partial y}, \\
\frac{\partial z}{\partial t} + \frac{\partial z}{\partial x} + \frac{\partial \zeta}{\partial y} + \frac{\partial \zeta}{\partial y} &= 0,
\end{align*}
\]

where \(u(x,y,t)\) and \(v(x,y,t)\) are the depth-averaged velocity components in the \(x\) and \(y\) directions, respectively, \(z(x,y,t)\) is the water-surface elevation measured from the undisturbed water surface, \(h(x,y)\) is the water depth also measured from the undisturbed water surface, \(g\) is the constant gravitational acceleration, and \(\gamma\) is the bottom friction coefficient given by

\[
\gamma = g \sqrt{u^2 + v^2} C_z^2 H,
\]

where \(H(x,y,t) = h(x,y) + z(x,y,t)\) is the total water depth, and \(C_z\) is the Chezy friction coefficient. The value of the Chezy number is calculated from the expression as in [18]:

\[
C_z = \frac{\phi}{n} R^{1/6},
\]

where \(\phi\) is a unit conversion coefficient which equals 1.0 if the SI units are used, \(n\) is a coefficient based on the channel character, and \(R\) is the average depth. For Mobile Bay, we chose \(n = 0.015\) for a mud bed and the average depth is \(R = 3\) m. The resultant Chezy number is 80. For Calcasieu Lake, the value is calculated as 71.3.

In Eq. (1), the vertical shear stress effects are represented by the bottom friction terms. Another part of the vertical shear stress effects that are caused by the surface wind shear is neglected. This is because the Calcasieu Lake area and the Mobile area are relatively calm areas shown from the wind statistics data from www.windfinder.com. The average wind speed in the Calcasieu Lake area is 9 mph(4 m/s), and the chance of wind is 24%. The average wind speed in the Mobile area is 8 mph(3.5 m/s), and the chance of wind is 16%. So in the majority of time, it is very calm in both areas. Our results agree well with the actual measurement data in Calcasieu Lake for May 2009 when there is no major storm, which indicates that the wind effect is negligible for simulating these areas under their regular weather conditions. The horizontal shear stress effects are also neglected because they are small in comparison with the vertical shear stress effects. Coriolis forces are not considered in this model, although the Rossby Number in the cases studied here are in the range of 0.1–0.5. In this range of the Rossby number, it is on the borderline that the Coriolis force would be considered. For that reason, we implemented the Coriolis forces term in one of the study cases, and the results, however, only had negligible differences from those without the Coriolis force effect. Therefore, Coriolis forces are neglected in this study.

Casulli [5] proposed a semi-implicit algorithm to solve Eq. (1). By substituting the two discretized momentum equations into the surface elevation equation, a pentadiagonal system for \(z\) can be obtained which is diagonally dominant and positive definite. The performance of this method is further improved when an ADI technique is combined: at the first half time step, the \(u\)-equation is substituted into the \(z\)-equation; at the second half time step, the \(v\)-equation is substituted into the \(z\)-equation. Such a procedure results in two sets of simpler, linear tri-diagonal systems which can be efficiently solved. These two equation sets for the simulation cases presented in this paper can be expressed as:

(1) Mobile River
(2) Tensaw River
(3) Blakeley River
(4) Dog River

Fig. 2. Water depth and computational domain of Mobile Bay (unit in meters).
Step 1:

\[
\left(1 + \gamma^k_{i,j} \Delta t\right) u_{i,j}^{k+1/2} = F u_{i,j}^{k+1/2} - \frac{\Delta t}{\Delta x} \left( z_{i,j}^{k+1/2} - z_{i,j}^{k-1/2} \right)
\]

and

\[
\gamma^k_{i,j} - \frac{\Delta t^2}{2 \Delta y^2} \left[ \frac{H^k_{i,j+1/2} + H^k_{i,j-1/2}}{2} \left( z_{i,j}^{k+1/2} - z_{i,j}^{k-1/2} \right) \right] - \frac{H^k_{i,j+1/2}}{1 + \gamma^k_{i,j} \Delta t} \left( z_{i,j+1}^{k+1/2} - z_{i,j}^{k-1/2} \right) - \frac{H^k_{i,j-1/2}}{1 + \gamma^k_{i,j} \Delta t} \left( z_{i,j-1}^{k+1/2} - z_{i,j}^{k-1/2} \right) \]
\[
= z_{i,j}^{k+1/2} \left[ \frac{H u_{i,j+1/2}^{k+1/2}}{1 + \gamma^k_{i,j} \Delta t} - \frac{H u_{i,j-1/2}^{k+1/2}}{1 + \gamma^k_{i,j} \Delta t} \right] + \frac{\Delta t}{2 \Delta y} \left[ \frac{H^k_{i,j+1/2} F v_{i,j+1/2}^{k+1/2} - H^k_{i,j-1/2} F v_{i,j-1/2}^{k+1/2}}{1 + \gamma^k_{i,j} \Delta t} \right].
\]

Step 2:

\[
\left(1 + \gamma^k_{i,j} \Delta t\right) v_{i,j}^{k+1/2} = F v_{i,j}^{k+1/2} - \frac{\Delta t}{\Delta y} \left( z_{i,j+1}^{k+1/2} - z_{i,j}^{k-1/2} \right)
\]

and

\[
z_{i,j+1/2} - \frac{\Delta t^2}{2 \Delta x^2} \left[ \frac{H^k_{i+1/2,j} + H^k_{i-1/2,j}}{2} \left( z_{i,j+1/2}^{k+1} - z_{i,j-1}^{k+1} \right) \right] - \frac{H^k_{i+1/2,j}}{1 + \gamma^k_{i,j} \Delta t} \left( z_{i+1,j}^{k+1} - z_{i,j}^{k+1} \right) - \frac{H^k_{i-1/2,j}}{1 + \gamma^k_{i,j} \Delta t} \left( z_{i-1,j}^{k+1} - z_{i,j}^{k+1} \right) \]
\[
= z_{i,j+1/2} \left[ \frac{H u_{i+1/2,j}^{k+1} - H u_{i-1/2,j}^{k+1}}{1 + \gamma^k_{i,j} \Delta t} \right] + \frac{\Delta t}{2 \Delta x} \left[ \frac{H^k_{i+1/2,j} F v_{i+1/2,j}^{k+1} - H^k_{i-1/2,j} F v_{i-1/2,j}^{k+1}}{1 + \gamma^k_{i,j} \Delta t} \right].
\]

In the above expressions, the indexes \( i, j \) and \( k \) denote the discretized points on the \( x, y \) and \( \tau \) coordinates, respectively. The appearances of the half steps in the spatial indexes (\( k \) and \( j \)) are due to the staggered grid mesh used in the computation which will be explained later. The operator, \( F \), is an explicit difference operator for convective terms. Its form will also be discussed later. The two water elevation equations in the two steps are both symmetrically tri-diagonal, and diagonally dominant with positive elements on the main diagonal and negative ones elsewhere. Therefore they have unique solutions which can easily be determined by a direct method. Once the elevation values are determined, the new values of \( u \) and \( v \) velocities can be easily evaluated explicitly by using the discretized momentum equations in the two steps.

An Eulerian–Lagrangian approach for calculating convection terms was discussed in [5] which allowed large time steps. In [4], a blended first- and second-order upwind approximation was employed and used in the simulation here. The major advantage of the Eulerian–Lagrangian approach is in the numerical stability requirement for the time step, \( \Delta t \). With the Eulerian–Lagrangian approach, the scheme is unconditionally stable. The restrictions on the time step are thus only from the consistency requirement. For the first-order upwind scheme, Casulli [5] has proved, using the von Neumann stability analysis, that the stability condition is

\[
\Delta t \leq \frac{|u|}{\Delta x} + \frac{|v|}{\Delta y}.
\]

However, because of the fast convergence we observed in the steady-state simulations, the stability condition, Eq. (6), does not seem to be restrictive, and therefore the upwind scheme was chosen for the simulations presented in this paper.

Staggered grids have been used in these methods where each cell is numbered at its center with indices \( i \) and \( j \). The discrete \( u \) velocity is then defined at half integer \( i \) and integer \( j \); \( v \) is defined at integer \( i \) and half integer \( j \), and \( z \) is defined at integer \( i \) and integer \( j \). Simple two-point averages from closest scalar grid points are also used to obtain all the required values on the staggered grid points. Therefore the scheme is second-order accurate in space at its best, even with higher-order accurate convection terms. As discussed previously, the grid mesh is a rectangular Cartesian mesh.

A mechanism to automatically identify wetting and drying of computational cells was implemented [4,6,19]. This simplifies the computer algorithm in that the surface elevation equation can be applied to all points throughout the domain. The presence of islands and other permanently dry areas as well as tidal flats can be accounted for appropriately and automatically. The implementation is based on the fact that a negative value for the total depth, \( H \), is physically meaningless. Therefore in the computation, \( H \) is defined as

\[
H = \max(0, h + z).
\]

A resulting zero value of the total depth \( H \) causes the respective friction factor \( \gamma \) (defined in Eq. (2)) to be infinity. Accordingly, the corresponding velocity \( u \) or \( v \) is forced to vanish.

The Dirichlet-type boundary conditions are specified at the inlet boundary for velocities. On the dry cells zero velocities are specified, and on the wet cells the inflow velocity values are calculated based on the river flow data of the Calcasieu River in the north of Calcasieu Lake, and four major rivers around Mobile Bay. At the outlet boundary, the boundary conditions are

\[
\frac{\partial z}{\partial \tau} = 0,
\]

and

\[
\frac{\partial H}{\partial \tau} = 0.
\]

where \( n \) and \( \tau \) are the normal and tangential directions of the outflow boundary, respectively. The water elevation boundary conditions are obtained by linear extrapolation from the water elevations at the interior nodes, except at the outlet where the tidal elevations are specified.

3. Results discussion

In both Calcasieu Lake and Mobile Bay simulations, the coastline and water depth data were extracted from the geographical data system produced by NGLI [14]. A minimum-distance criterion was used as a search algorithm to obtain the depth data on the grid points. Such an algorithm resulted in a “saw-tooth” type boundary grids. As in the discussions of the errors issues related with the “saw-tooth” type grids in [20], the effects of the errors are restricted to local areas adjacent to the boundaries. When these small regions are of interest, nested grids [21] will be used for these regions in the future research. Therefore, for the purpose of considering the overall circulation, the errors are negligible.

3.1. Calcasieu Lake

The computational domain described in Fig. 1 is between latitudes 29.710 to 30.123, and longitudes -93.445 to -93.207, which is a 25.8 km by 44.8 km rectangular domain. The grid number is 100 by 174, with \( \Delta x = 260.648 \) m and \( \Delta y = 258.831 \) m. A finer grid mesh with double the number of grids in each dimension was tested, and no significant changes were shown in the steady-state results, which justified the existence of a grid-independent solution with the current grid resolution. A steady-state solution was
first obtained with an average tide height of 0.189 m. Average inflow for Calcasieu River is 178 m$^3$/s. The river inflow data used in this paper are from [22]. The time step used in the steady-state solution can reach as large as 290 s to satisfy the stability requirement in Eq. (6). The converged solution, which required the velocity and outflow to become steady (converged to less than $10^{-4}$) and inflow and outflow to be balanced (less than 0.3% difference), was reached after about 1000 time steps. In the unsteady simulation, a more conservative time step of 60 s was set for the Calcasieu Lake case to obtain time accuracy of the solution as well as to satisfy the stability condition.

The unsteady ocean tidal conditions of Gulf of Mexico was implemented in the unsteady simulations of the Calcasieu Lake circulation. A set of ocean-surface elevation data from NOAA [17] is available, thus used as the outlet boundary condition of the simulation. The time interval of two consecutive time series data sets is 6 min. The period covered by the data is from May 1st to May 31st 2009. The corresponding changes of the water-surface elevation inside Calcasieu Lake were monitored and recorded during the simulation. The actual water-surface elevations were also measured inside the lake by SW LA NWR Complex C-21 Hwy 384 Station located at the northeast corner of the lake [23]. Therefore, the simulation results at the same location can be compared with the measurement data to validate the accuracy of the simulation. Fig. 3 is the comparison, where (a) is for the first 15 days and (b) is for the next 15 days of the month. In this figure, the actual ocean elevation curve is also included. The simulation predictions match very well with the measurement, particularly in the oscillatory unsteady behavior. This agreement thus validates the accuracy of the unsteady simulation. The periods of both simulation results and measurement data show a little delay comparing with the ocean tidal period, which is due to the time delay for the tidal waves to travel from ocean to the lake.

Another interesting observation from Fig. 3 is that the oscillating magnitude of the lake wave is not much smaller than that of the ocean tidal wave. It indicates that the influence from Gulf of Mexico is great, and that the shallow lake and surrounding wetlands do not have significant effect on avoiding the ocean flood and protecting the relative inland cities, such as Lake Charles. This may explain the reason of severe floods caused by recent hurricanes. The reason is that the deepening and widening ship channel allows ocean water to flow more freely into the lake, making the lake behave more “ocean-like”.

The effect of Calcasieu Ship Channel on lake hydrodynamics is then investigated. A case using original geometric data without the ship channel is simulated using the same ocean elevation data. The result is compared to the result with ship channel, and plotted in Fig. 4. It is clear that without the ship channel, less oscillation of the water-surface elevation can be founded although the average water level is higher than that with ship channel. The lower oscillation can be beneficial when an event such as a storm surge caused by a hurricane occurs: the water level in the lake will not rise as high. Without the ship channel, the fact that the response of water level change in the lake is not sensitive to the ocean level change enables Calcasieu Lake act as a buffer between the ocean and the inland cities.

Fig. 5 is the water-surface elevation contours at low tide and high tide in the lake, comparing cases with and without ship channel. It needs to be noted that at high tide in the lake, the lake elevation is higher than the ocean elevation due to the delay, and vice versa. Sensitive response of lake water level to ocean water level can also be observed for the case with the ship channel. In addition, the response is greater in the northeast side of the lake, where the
water-surface elevation is lower than the rest of the lake at low tide and is higher at high tide. The velocity vectors and the instantaneous streamlines (the solid line) at low tide and high tide in the lake are plotted in Fig. 6, for the case with ship channel. The streamlines show the paths of water from Calcasieu River at two tidal conditions. The hydrodynamics of the lake is revealed in the figure. When it is low tide in the lake, ocean water flows into the lake through the ship channel through the Calcasieu Pass. Fresh water input from the Calcasieu River is stopped at the entrance of the Calcasieu Pass, which causes complex flow structures and circulations. This is also evidenced by the streamline which is stopped near the entrance of the Calcasieu Pass. On the contrary, when it is high tide in the lake, lake water flows out and the flow structure is less complicated. The streamline started from

Fig. 5. Water-surface elevation contour comparison for Calcasieu Lake case (unit in meters). (a) With Calcasieu Ship Channel, low tide in the lake, (b) with Calcasieu Ship Channel, high tide in the lake, (c) without Calcasieu Ship Channel, low tide in the lake, (d) without Calcasieu Ship Channel, high tide in the lake.
Calcasieu River shows a different path comparing to that at low tide in the lake. In both tidal conditions, the flow near the Calcasieu Pass is much stronger than anywhere else in the lake.

Streamlines in Fig. 6 show the instantaneous paths at particular moments, but are unable to show particle trajectories in a period of time. Particle tracers for the Calcasieu Lake case for the first 10-day period of May 2009 are plotted in Fig. 7. These particle tracers are for non-reactive, non-diffusive particles that passively follow the flow. In this paper, the particle tracking has been done by a flow visualization commercial software. The transient particle path has been calculated using the particle path integration scheme for unsteady data which includes convection due to the velocity field. The particle position at a new time step is calculated by the velocity field at that time and the previous particle position and velocity. The scheme is a fourth-order implicit multi-step scheme [24]. The start-up procedure for this scheme is third-order, so is the global error. Four particles are evenly placed along the Calcasieu Ship Channel to track the transport of the particles that can be used as representations of pollutants or sediments. The dots in Fig. 7 indicate the original locations of the four tracer particles. The particles were released at the beginning of the simulation time which is May 1st. The color of the particle path line indicates the time in seconds. From Fig. 7, it is clear that the trajectories of the upstream particles differ greatly from those of the particles in the downstream. All particles follow a wavy path due to the tidal motion from the ocean. The upstream particles struggle to flow south, because the flow is very slow in the area which is shown in Fig. 6. The downstream particles flow fast and race to the ocean although the path is more complicated than the upstream particles. The complicated trajectories are indicated by the complex flow pattern near the exit of the lake due to the ocean tide. The downstream particles have faster speed, indicating a fast removal rate of pollutants or sediments. According to the simulation, it takes about 2 days for the downstream particle to enter the ocean, and about 7 days for the second particle from the bottom. The results also indicate that the upstream ship channel has more sedimentation and pollution concentrations than the downstream of the channel, which conforms with the actual situation. About 4 millions cubic yard of sediments were removed every two years in the ship channel above the Calcasieu Pass, and very few sediments were observed in the Calcasieu Pass which is at the downstream of the ship channel [25].

3.2. Mobile bay

The computational domain of Mobile Bay described in Fig. 2 is between latitudes 30.708 and 30.099, and longitudes −88.162 and −87.697, which is a 50 km by 65 km rectangular domain. The grid number is 140 by 188, with $\Delta x = 360.112$ m and $\Delta y = 347.59$ m. The grid-independence check was also performed to ensure a grid-independent solution with the current grid resolution. A steady-state solution was first obtained with an average tide height of 0.189 m. Average inflow for Mobile River, Tensaw River, Blakeley River and Dog River are 1807 m$^3$/s, 416 m$^3$/s, 142 m$^3$/s and 0.631 m$^3$/s, respectively. Since the average flow rate of Dog River is very small, it is assumed zero as far as average flow rate is concerned. The effects of Dog flow were tested when the flow rate was at its maximum of 144 m$^3$/s. The results, in comparison with the simulation results of zero inflow rate of Dog River, showed that there were no significant effects, at least within its maximum inflow rate. The river inflow data used in this paper are from [3] which were based on the US Geological Survey from
US Department of Interior. The time step used in the steady-state solution can reach as large as 418 s to satisfy the stability requirement in Eq. (6). The converged solution was reached after 1200 time steps. The steady-state simulation result is shown in Fig. 8 for the velocity vectors and streamlines.

The tide equation is assumed to have the format of a sinusoidal wave,

\[ z(t) = 0.189 + 0.275 \sin \left( \frac{2\pi t}{T} \right), \tag{10} \]

where the unit of \( z \) is meter, and the period \( T \) is 24 h. These data are based on the National Ocean Service observation data at the Fort Gaines station in Dauphin Island, AL. The tidal simulation cases started at time zero, using the steady-state solution with the average elevation as the initial solution. The minimum time step in the unsteady cases was calculated to be about 100 s, according to Eq. (6) for the current grid size. A more conservative value of 40 s for the time step was used for all the unsteady computation to obtain time accuracy of the solutions as well as to satisfy the stability condition.

Fig. 9a is the velocity vectors and the instantaneous streamlines at the trough of the ebb, when the tide is at the lowest point with tide height of \( -0.086 \) m, and Fig. 9b is the water elevation (for the quantity of \( z(x,y,t) \)) contours for the same case. The high velocity and high elevation-gradient regions at the upper side of the figures represent the average inflow effects from the three rivers: Mobile River, Tensaw River and Blakeley River. The average inflow of Dog River is zero in this case, as stated previously. The high velocity and high elevation-gradient area at the lower portion near Dauphine Island is where the main pass of the ship channel is located. It can be seen that most outflow of Mobile Bay is going out through the main pass during the ebb. The instantaneous streamline pattern indicates that the transport is facilitated all the way from the river inflow to the main pass.

Fig. 10a and b are for the high tide case when the tide is at its peak point of 0.464 m. Other conditions are the same as those in the ebb case. Again, the high velocity and high elevation-gradient region is at the main pass. The difference is at this time, the velocity is inward to the bay area. The transport is from the ocean to inside the bay. At the top of Fig. 10a, it shows that some of the inflow from Mobile River may go to Blakeley River because of the high tide effects, which causes negative flow rate at the Blakeley River site. Such a phenomenon is actually compatible with some measured data [3] which showed reversed inflow in Blakeley River.

Figs. 11 and 12 are particle tracers for the steady-state condition and the tide condition, respectively. These particle tracers are the same type of tracers as in Fig. 6. Due to the non-reactive, non-diffusive characteristics of the particles tracers, the particle tracer lines in the steady case, Fig. 11, are the same as the streamlines in Fig. 8, the fact that validates the particle tracer algorithm used here. It can be seen that for the same particle to be transported out of Mobile bay starting from dumping in the rivers such as Mobile River, a time period of 15 days (after 15 periods of simulations of the ebbs and high tides in Eq. (10)) is required for the steady-state case, while a period of 12 days is required for the tide case. This means that the tide condition helps the transport of contaminants out of Mobile Bay, although the particles have to follow a wavy path due to the tidal motion from the ocean. Another important fact shown by the particle tracer lines is that...
Fig. 9. Flow and elevation at the trough of the ebb for Mobile Bay case. (a) Velocity vectors and streamlines, (b) elevation contours (unit in meters).

Fig. 10. Flow and elevation at the peak of the high tide for Mobile Bay case. (a) Velocity vectors and streamlines, (b) elevation contours (unit in meters).
the dumped contaminants near Mobile River and Tensaw River (near particle liners at the center of Mobile Bay) are much easier to be transported out of Mobile Bay than those from Blakeley River which is close to the side boundary of the bay with relatively low flow rate. The transport of the particles originated from Dog River would be insignificant because of its almost zero flow rate.

Interesting differences can be found when comparing the particle paths of Calcasieu Lake to those of Mobile Bay. The path is less complicated in Mobile Bay than Calcasieu Lake. The possible reason could be the geometric difference of the two water system. The shapes of both system are similar but the exit area. Mobile Bay is relatively open to the Gulf of Mexico while Calcasieu Lake exits to the ocean through a narrow Calcasieu Pass. The narrow exit causes (1) faster flow, (2) more complex flow near the exit, and (3) less water flowing into and out of the lake comparing to the open exit of Mobile Bay. Faster flow results faster removal of particles downstream of the channel. The more complex flow pattern leads to more complex trajectories of particles near the exit. And less water flow causes the stagnant upstream particle paths.

4. Conclusions

The hydrodynamic model presented in the paper has provided a tool to simulate Calcasieu Lake and Mobile Bay circulations using a robust numerical scheme on a Cartesian grid mesh. This numerical model can accommodate complicated geometric features around the two water systems. Transport properties and water elevations have been simulated that include influences from the major rivers and unsteady tide motions. The results are compared to the measurement data, and the good agreement shows the accuracy of the simulations. The geometric effects on the hydrodynamics and transport have been shown. In Calcasieu Lake, the presence of the ship channel leads to the increase of water exchange between the ocean and the lake, and causes a sensitive response of lake water level to the change of ocean water level. The narrow exit passage leads to complex flow patterns when ocean water flowing into the lake. Using passive particle tracers following the flow, it can be shown that the trajectories of particles at the upstream and downstream of the ship channel are very different. Overall, it takes very long time for upstream particles to flow out of the lake, resulting in excessive sedimentation in the ship channel. In Mobile Bay, the high tide and ebb cases have shown the effects of tides on the Mobile Bay circulation and transport. Using passive particle tracers, it can be shown that the period for particles to be transported out of Mobile Bay depends on the tide conditions. A typical period is about two weeks for a particle to pass through the bay to the ocean starting from the river area at the top of Mobile Bay. Regions near Mobile River and Tensaw River have better transport rates than the regions close to side boundaries in the bay. In addition, the trajectories of particles in Calcasieu Lake and Mobile Bay differ significantly due to different exit geometries of the two systems. This simulation tool and its results can be used to study the scenario of sedimentation and pollution in the investigated areas, and can possibly be extended to other similarly structured shallow-water areas.

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