Appendix A  Variances of the Modified GMC Method

The variance-covariance matrix of the averaging method is discussed in detail by Yun (2008). The same technique can be applied for the modified GMC method with slight modification.

The variance-covariance matrix of the normalized regression coefficients is computed as $\Sigma_{b^*} = W \Sigma_{B^0} W'$ where $W$ is a weight matrix and $\Sigma_{B^0}$ is a reformatted variance-covariance matrix of the original regression coefficients ($\Sigma_B$). The weight matrix $W$ for the averaging method used in this study is defined as:

$$W = \begin{bmatrix}
1 & (1/J) \cdot 1_{1 \times J} & 0_{1 \times K} & (1/L) \cdot 1_{1 \times L} \\
0_{J \times 1} & M_{J \times J} & 0_{J \times K} & 0_{J \times L} \\
0_{K \times 1} & 0_{K \times J} & I_{K \times K} & 0_{J \times L} \\
0_{L \times 1} & 0_{L \times J} & 0_{L \times K} & M_{L \times L}
\end{bmatrix}$$

where $M_{J \times J} = I_{J \times J} - (1/J) \cdot 1_{J \times J}$ in which $J$ represents for the number of education variables (i.e., four) and 0 and 1 are a matrix of zeros and a matrix of ones, respectively. $I_{K \times K}$ refers to the $K$ by $K$ identity matrix for two age variables. $L$ represents the number of marriage variables (i.e., two).

The reformatted variance-covariance matrix of the regression coefficients ($\Sigma_{B^0}$) is attained by adding zero vectors to the variance-covariance matrix of the original regression coefficients, as follows:

$$\Sigma_{B^0} = \begin{bmatrix}
\sigma_a^2 & 0 & \Sigma_{a,b'_j} & \Sigma_{a,b'_k} & 0 & \Sigma_{a,b'_L} \\
0 & 0 & 0_{1 \times (J-1)} & 0_{1 \times (K-1)} & 0 & 0_{1 \times (L-1)} \\
\Sigma_{b_j,a} & 0_{(J-1) \times 1} & \Sigma_{b_j,b'_j} & \Sigma_{b_j,b'_k} & 0_{(J-1) \times 1} & \Sigma_{b_j,b'_L} \\
\Sigma_{b_K,a} & 0_{(K-1) \times 1} & \Sigma_{b_K,b'_j} & \Sigma_{b_K,b'_k} & 0_{(K-1) \times 1} & \Sigma_{b_K,b'_L} \\
0 & 0 & 0_{1 \times (J-1)} & 0_{1 \times (K-1)} & 0 & 0_{1 \times (L-1)} \\
\Sigma_{b_L,a} & 0_{(L-1) \times 1} & \Sigma_{b_L,b'_j} & \Sigma_{b_L,b'_k} & 0_{(L-1) \times 1} & \Sigma_{b_L,b'_L}
\end{bmatrix}$$
where \( \sigma_a^2 \) is the residual variance and \( \Sigma \) is a partial covariance matrix. For example, \( \Sigma_{b_K,b_J} \) is a covariance matrix between education coefficients \( b_J \) and age coefficients \( b_K \); 0 refers to a zero matrix.

For the modified GMC method, everything except the weighting matrix \( W^* \) is the same as the averaging method. The weighting matrix needs to be rebuilt as follows:

\[
W^* = \begin{bmatrix}
1 & G_{1 \times J} & 0_{1 \times K} & G_{1 \times L} \\
0_{J \times 1} & N_{J \times J} & 0_{J \times K} & 0_{J \times L} \\
0_{K \times 1} & 0_{K \times J} & I_{K \times K} & 0_{J \times L} \\
0_{L \times 1} & 0_{L \times J} & 0_{L \times K} & N_{L \times L}
\end{bmatrix}
\]

where \( G_{1 \times J} \) refers to a \( 1 \times J \) column vector of grand means for education variables and \( G_{1 \times L} \) refers to a \( 1 \times L \) column vector of grand means for marriage variables. \( N_{J \times J} \) denotes \( I_{J \times J} - 1_{J \times J} \cdot D_{J \times J} \) where \( D_{J \times J} \) refers to a diagonal matrix converted from the column vector of the grand means of education.

The new coefficients for the modified GMC method, \( (B^*) \) can be obtained by taking diagonal values of \( W^*B \). The variance-covariance matrix of the new coefficients is computed as \( \Sigma_{b^*} = W^*\Sigma_B W^{*\prime} \). The variance of any predicted value \( \hat{X}B \) is estimated as \( X \Sigma_B X' \). Likewise, the variance of \( \bar{X}B^* \) can be estimated as \( \bar{X} \Sigma_{b^*} \bar{X}' \).

Once the variance-covariance matrix of the coefficients of the modified GMC method is computed, the standard errors for the detailed decomposition can be obtained relatively easily. Using matrix notation, D1B in Equation 2 can be expressed as \( \bar{X}B \hat{B}_{D1B}^* \). The variance of \( \bar{X}B\hat{B}_{D1B}^* \) is therefore calculated by \( \bar{X} \Sigma_{C^*} \bar{X} \), where \( \Sigma_{C^*} \) is an addition of the two variance-covariance matrices of coefficients, \( \Sigma_{b^*_W} + \Sigma_{b^*_B} \). The variances of D2 are estimated in the same way. As we estimate the variances of the decomposed components, the t-tests of the estimated decomposition components can be conducted simply; for example, by \( t_{D1B} = \frac{D1B}{\sigma_{D1B}} \). Note that here we assume the \( \bar{X} \)'s to be fixed. if \( \bar{X} \)'s are stochastic, the estimation of variances should include the variance of \( \bar{X} \) and the interaction effects (Lin 2007; Jann 2008).
References

