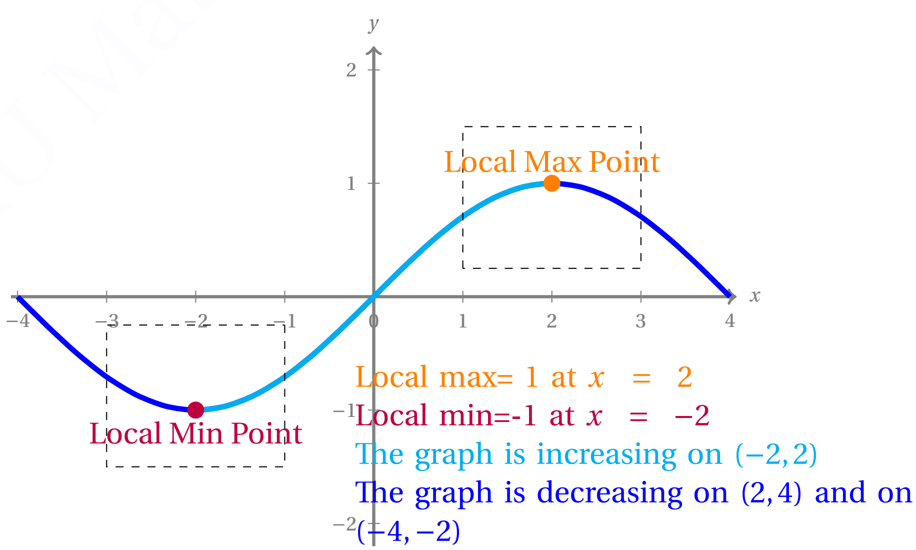
# 1.3: Rate of Change, Behavior of the Graph

## Local Extrema, Increasing and Decreasing Graphs.

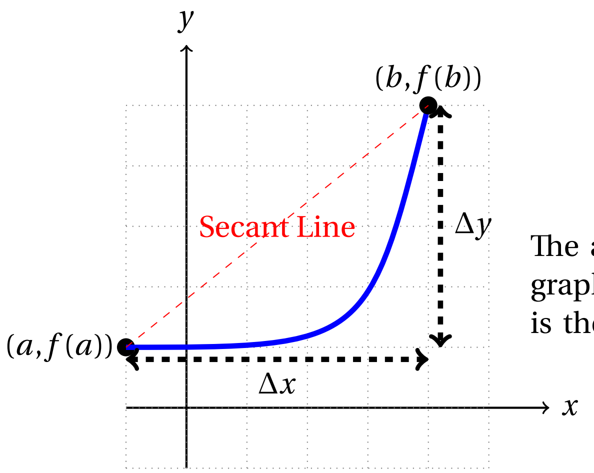
- Function $f$ is increasing on an **open** interval $I = (a, b)$ if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$. (That is, the output increases as the input variable increases, going from left to right.)
- Function $f$ is decreasing on an **open** interval $I = (a, b)$ if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$. (That is, the output decreases as the input variable increases.
- A value of the input where a function changes from increasing to decreasing as the input variable increases (as we go from left to right) is called a local maximum.(plural maxima) A local maximum is an input value $c$ such that for all values $x \neq c$ in an interval $(a, b)$ containing $c$, $f(c) \geq f(x)$.
- A value of the input where a function changes from decreasing to increasing as the input variable increases is called a **local minimum**. A local minimum is an input value $c$ such that for all values $x \neq c$ in an interval $(a, b)$ containing $c$, $f(c) \leq f(x)$.
- The local maximum or minimum values of $f$ are also called extremum values.
- A local maximum point may not be the highest point on the graph. The highest point on the graph, if it exists, is called the absolute maximum point of the graph. Similarly a local minimum point may not be the lowest point on the graph. The lowest point of graph, if it exists, is called absolute minimum point of the graph.
- **Examples:**




Now, you can complete Problems 1 and 2.

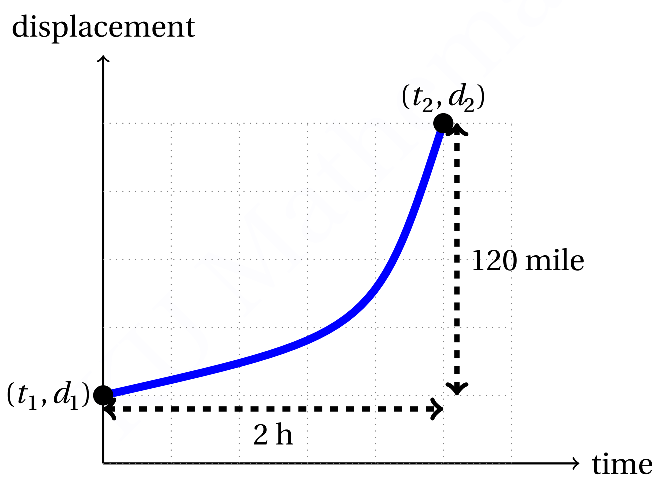
## Average Rate of Change

- The average rate of change for function $f(x)$ between $x = a$ and $x = b$ is the ratio of change in $y$-value to the change in $x$-value.
  $\dfrac{\text{Change in } y}{\text{Change in } x} = \dfrac{\Delta y}{\Delta x} = \dfrac{f(b) - f(a)}{b - a}$
- The slope of secant line between two points on the graph of $f$ is the average rate of change in $f$ between the two points. The direction of change is the direction of slope.




The average rate of change between two points on the graph (note that $y$-values come from the function) is the slope of the secant line between the two.

- Average velocity $= \dfrac{\text{displacement between points}}{\text{time elapsed}}$




The average velocity between two points is the ratio of displacement to time. For example, here the average velocity is $v_{\text{ave}} = \dfrac{120}{2} = 60\dfrac{mile}{hr}$

## Finding the Average Rate of Change in $f$ over $[a, b]$

(1) Find $f(b)$ and $f(a)$.

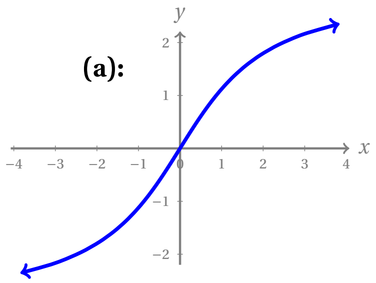
(2) Form $f(b) - f(a)$. Simplify and factor if possible.

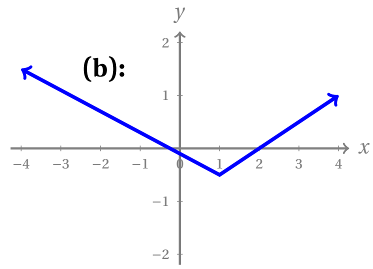
(3) Form $\dfrac{f(b) - f(a)}{b - a}$, simplify if possible.

(4) Note: If $b = a + h$, then look for a factor $h$ in $f(a + h) - f(a)$.

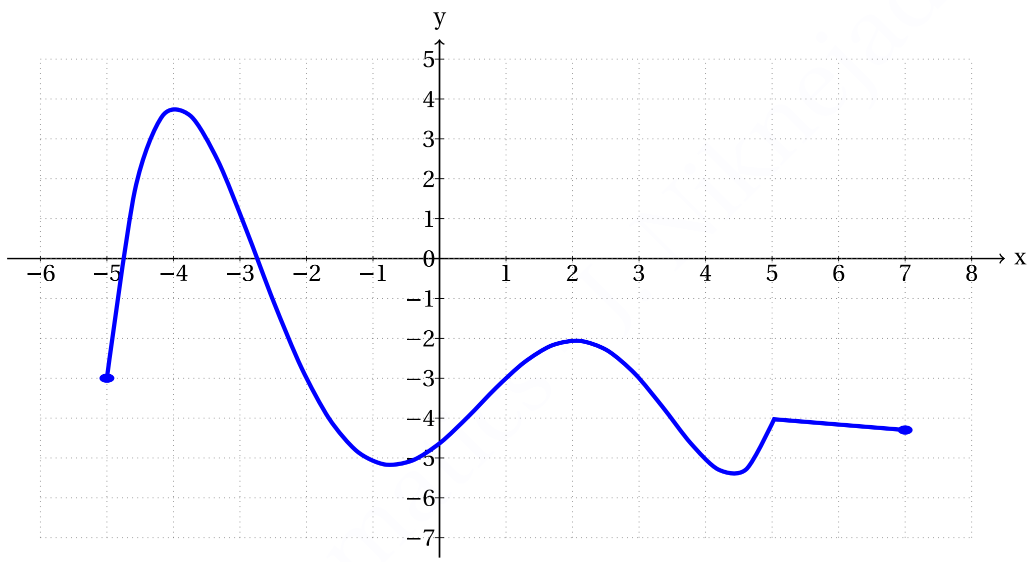
Now, you can complete Problems 3-5.

1. Find the interval(s) on which function is increasing and the interval(s) on which the function is decreasing for the following graphs:




2. The graph of function $f$ is given by:




(a) Find the values $f(-5)$, $f(0)$ and $f(2)$.

(b) What is the domain of the function?

(c) What is the absolute maximum value of the output variable? What is the absolute minimum value of the output variable?

(d) What is the range of the function?

(e) List all interval(s) where $f$ is increasing.

(f) List all interval(s) where $f$ is decreasing.

(g) List all interval(s) where $f$ is constant.

(h) What are the local maxima of the function?

(i) What are the local minima of the function.

3. Find the average rate of change of the function $g(z) = z^2 - 5$ on interval $[2, 5]$.

4. **Average Rate of Change** of a quadratic polynomial on $[a, a+h]$:

   (A) Let $f(x) = x^2 - 3x + 15$. Find average rate of change in $f$ on interval $[a, a+h]$ and simplify. (This is also called the **difference quotient** for $f(x)$. )

   (B) What do you think instantaneous rate of change is at $x = a$?

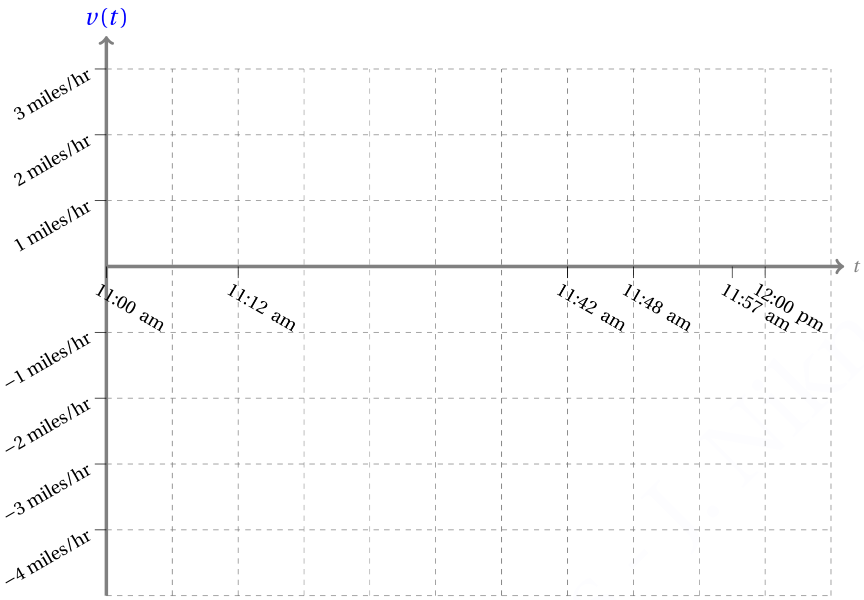
5. At time $t$ seconds, the position of a body moving along the $x$-axis is

$$s(t) = 2t^3 - 15t^2 + 24t \text{ meters.}$$

   What is average velocity (average rate of change of position) of the body during the time interval $[0, 1]$?

6. Bob wants to take a 1 hour break. At 11 am, Bob parks his car and walks south on Massachusetts Street at a constant **speed** of 3 miles/hr for 12 minutes when he arrives at Fuzzy's Tacos. He sits on the outside seating at Fuzzy's Tacos for 30 minutes with a friend, then he walks north at a **speed** of 3 miles/hr for 6 minutes when he runs into another friend and stops to talk to them for 9 minutes and then Bob starts walking north again at speed 4.5 miles/hr.

   (a) Graph $v(t)$, the velocity of Bob as a function of $t$ time in minutes during Bob's 1 hour break noting that velocity has direction.

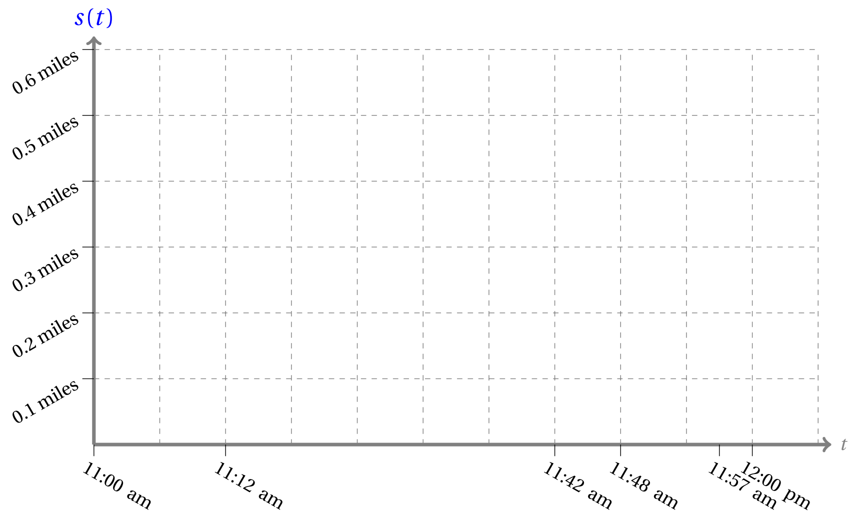



$$v(t) = \begin{cases} \_\_\_, & 11:00 < t \le 11:12 \\ \_\_\_, & 11:12 < t \le 11:42 \\ \_\_\_, & 11:42 < t \le 11:48 \\ \_\_\_, & 11:48 < t \le 11:57 \\ \_\_\_, & 11:57 < t \le 12:00 \end{cases}$$

   (b) Find how far Bob is from his car at the following times noting that velocity has direction. Remember to convert the time interval

| time | $s(t)$ miles |
|---|---|
| 11 : 12 am | |
| 11 : 42 am | |
| 11 : 48 am | |
| 11 : 57 am | |
| 12 : 00 pm | |

(c) Use the information in Part (b) to graph $s(t)$, the distance of Bob from his car at any time $t$ minutes noting that the distance changes linearly during each time interval.




(d) What is the interpretation of the slope of each line segment.

(e) What is the distance of Bob from his car at the end of his break?

(f) What is the total distance traveled by Bob, registered on his health app., during his break?

**Example Videos:**

1. https://mediahub.ku.edu/media/t/1_on01frtf
2. https://mediahub.ku.edu/media/t/1_wgwy9v48