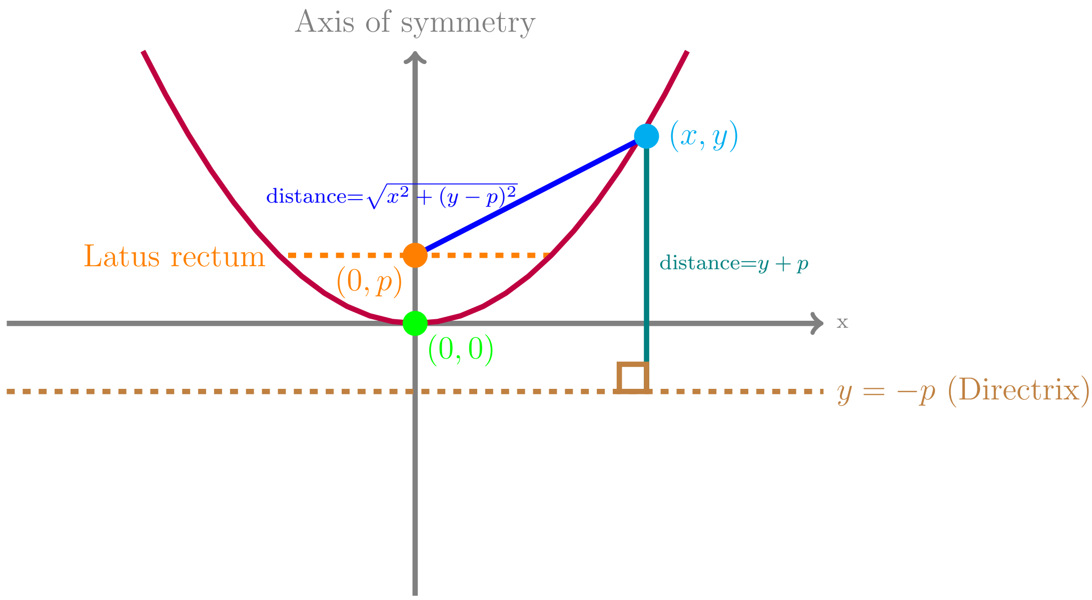
10.3: Parabolas

- Geometric definition: A parabola is the set of points that are equidistant from a point (**called the focus**) and a fixed line (**called the directrix**).
- Using the geometric definition to find a formula



Note that the distance between the vertex, $(0, 0)$, and the focus and the distance between the vertex, $(0, 0)$, and the directrix are equal. That is why focus is denoted by $(0, p)$ and directrix is denoted by $y = -p$. Now using these information we derive a closed form formula for the above parabola:

$$\sqrt{x^2 + (y-p)^2} = y + p$$

$$\underset{\text{Isolate one radical:}}{\Longrightarrow} \text{Already done.}$$

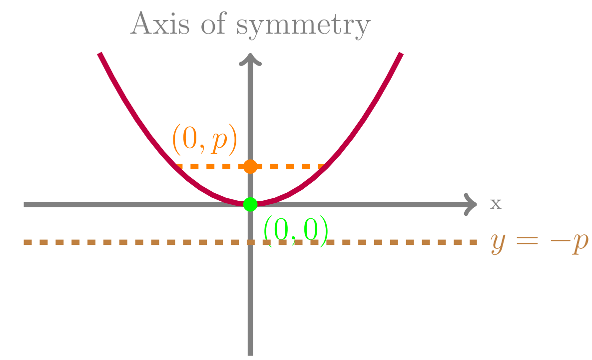
$$\underset{\text{Raise to Power 2:}}{\Longrightarrow} x^2 + (y-p)^2 = (y+p)^2$$

$$\underset{\text{Use binomial expansion and simplify:}}{\Longrightarrow} x^2 + y^2 - 2py + p^2 = y^2 + 2py + p^2$$

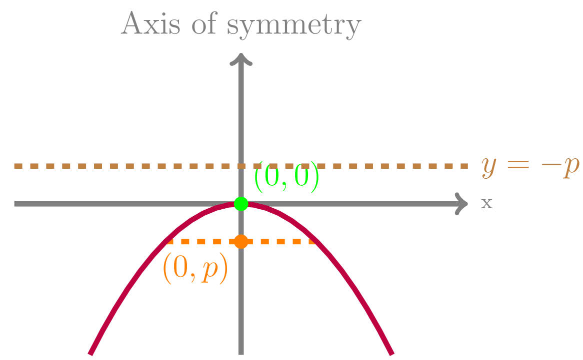
$$\underset{\text{Simplify:}}{\Longrightarrow} x^2 = 4py$$

- **Latus rectum:** The line segment through the focus, perpendicular to axis of symmetry with endpoints on the parabola is the latus rectum. The length of the latus rectum is called **focal diameter.** It can easily be seen that the length is $4|p|$: Plug in $y = p$ in the the closed form formula to get $x^2 = 4p^2$. Thus $x = \pm 2p$ are the two end points of the latus rectum. Therefore, the length is $4|p|$.
- **Applications:** The parabolic concave mirrors reflect any ray parallel to their axis of symmetry through their focus. They are used to gather energy at their focus. Also, if you need perfectly parallel light beams, place the light source in the focus of a concave mirror.

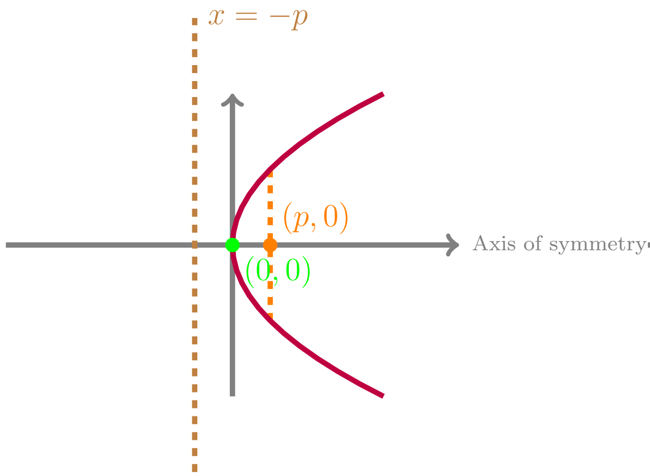
- Graphs of parabolas where axes are vertical or horizontal



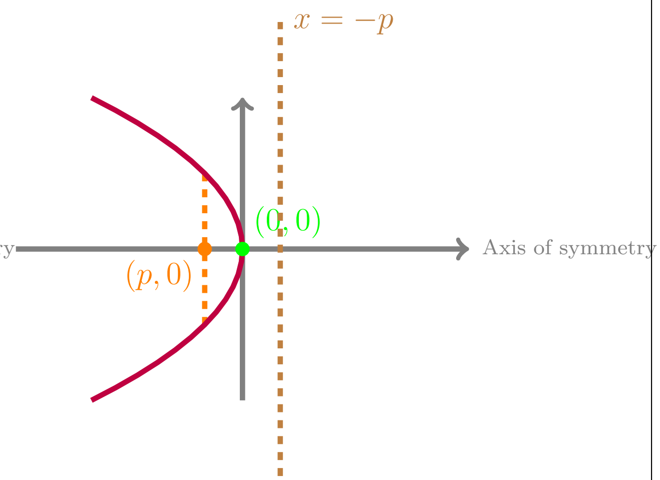
Standard Equation: $x^2 = 4py$, $p > 0$



Standard Equation: $x^2 = 4py$, $p < 0$



Standard Equation: $y^2 = 4px$, $p > 0$



Standard Equation: $y^2 = 4px$, $p < 0$

- How to find different parameters of a parabola using its equation:

1. If the equation is anything other than the above equations, reformat to one of the above.
2. If $x^2 = 4py$, then parabola opens along a vertical axis of symmetry. Otherwise, it opens up along a horizontal axis of symmetry.
3. Use the standard form to find p.
4. If the parabola opens up along a vertical axis of symmetry, treat it like the parabolas from Chapter 1. Find the transformations and vertex and so on.
5. If the parabola opens up along the horizontal axis, use transformations as well.

1. Find the focus, vertex, directrix and axis of symmetry of the following parabolas.

   (A) $x^2 = 9y$ (B) $x + 3 = -8(y - 5)^2$ (C) $8x^2 + 12y = 0$

2. Lynne is on an island and wants to start a fire on the beach using a parabolic concave mirror made by J. Portin & Company . The mirror's cross section follows the equation

   $$(x - 20)^2 = 18y.$$

   where should she put the tree bark and how should she place the rays of light?

   Fun activity: Watch a concave mirror at this link: https://ggbm.at/pwnxvpvj