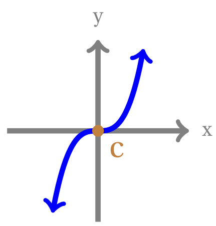
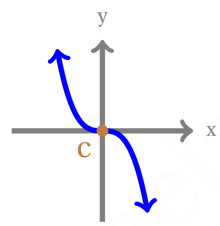
3.3 Power Functions and End Behavior

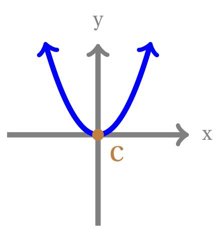
- The end behavior is the behavior of the graph of a function as the input decreases without bound and increases without bound.
- A **power function** is of the form: $f(x) = kx^p$ where k and p are constant. p determines the degree of the power function and both k and p determine the end behavior.



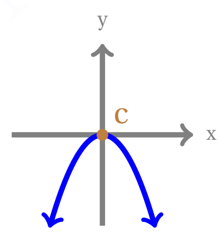
Power function, p: odd, $k > 0$
End behavior:
$y \to \infty$ as $x \to \infty$
$y \to -\infty$ as $x \to -\infty$



Power function, p: odd, $k < 0$
End behaviour:
$y \to -\infty$ as $x \to \infty$
$y \to \infty$ as $x \to -\infty$

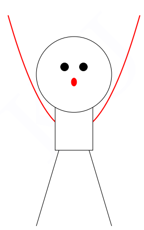


Power function, p: even, $k > 0$
End behaviour:
$y \to \infty$ as $x \to \infty$
$y \to \infty$ as $x \to -\infty$

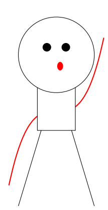


Power function, p: even, $k < 0$
End behaviour:
$y \to -\infty$ as $x \to \infty$
$y \to -\infty$ as $x \to -\infty$

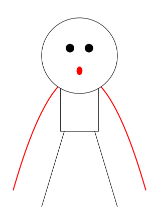
Math Dance for the End Behaviors:



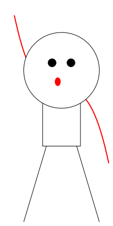
Degree: even, Leading coefficient: positive



Degree: odd, Leading coefficient: positive

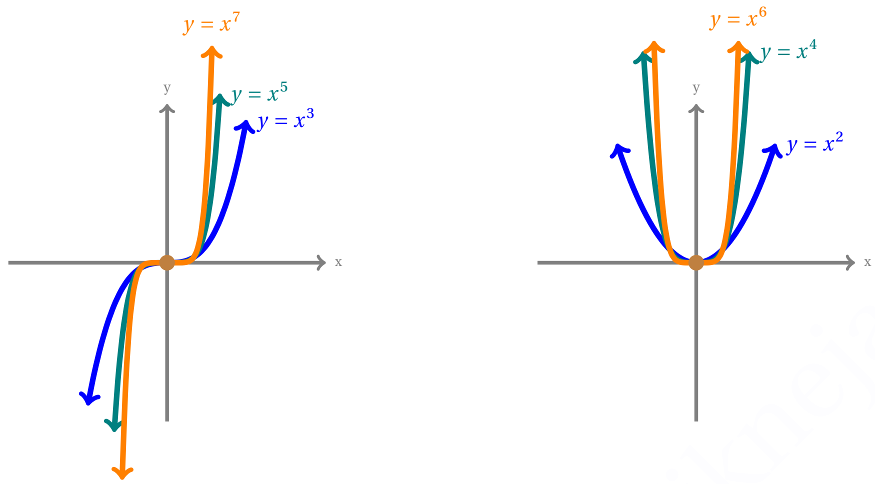


Degree: even, Leading coefficient: negative



Degree: odd, Leading coefficient: negative

Comparing Power Functions:



Polynomials

- A polynomial function is a function that consists of either zero or the sum of a finite number of non-zero terms, each of which is a product of a number, called the coefficient of the term, and a variable raised to a non-negative integer power.

- $$f(x) = \underbrace{\overset{\text{Leading coefficient}}{\overset{\downarrow}{a_n}}\, x^{\overset{\text{degree}}{\overset{\downarrow}{n}}}}_{\text{leading term}} \quad + \overset{\text{coefficient}}{\overset{\downarrow}{a_{n-1}}} x^{n-1} \quad \dots \quad + \overset{\text{coefficient}}{\overset{\downarrow}{a_1}} x \quad + \quad \underbrace{\overset{\text{Constant coefficient}}{\overset{\downarrow}{a_0}}}_{\text{constant term}}$$

- The leading term is the term containing the highest power of the variable. The leading coefficient is the coefficient of the leading term.

- A continuous function is a function whose graph has no break or jump. A smooth function is a function whose graph has no sharp corners.

- A polynomial is smooth and continuous. The end behavior of a polynomial is the same as its leading term. (Note that the leading term is a power function.)

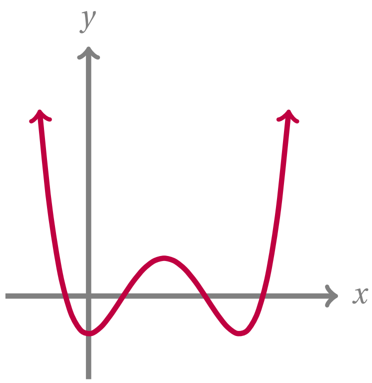
- The degree of a polynomial is the highest power of the variable that occurs in a polynomial.

- Turning point is the location at which the graph of a function changes direction. (a.k.a a local extremum) The number of turning points for a polynomial of degree n is $\leq n-1$. The number of real roots for a polynomial of degree n is $\leq n$.

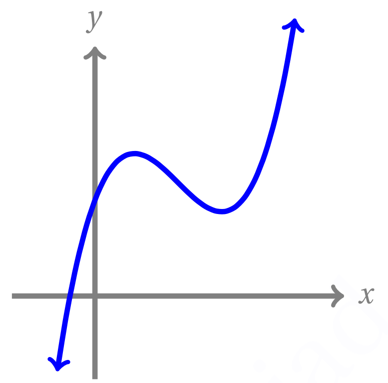
- Other forms: Often we refer to a polynomial in its factored form:

  $P(x) = a_n(x-x_1)(x-x_2)\dots(x-x_n)$. To find the standard form of the polynomial, foil all factors.

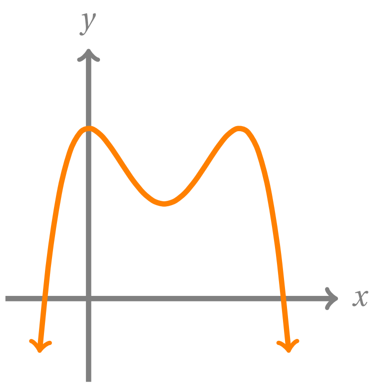
1. The following 4 graphs are graphs of polynomial functions. For each of the graphs, (a) Find whether the degree is odd or even. (b) Is the leading coefficient positive or negative? (c) What is the number of zeros? (d) What is the number of turning points?



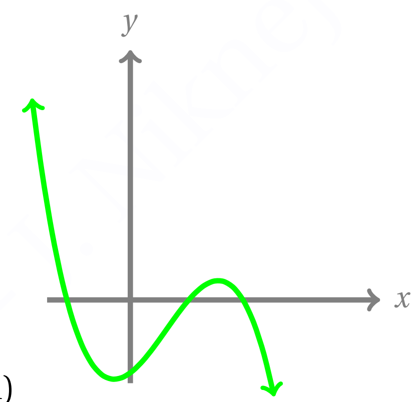
(a)



(c)



(b)



(d)

2. For each of the following functions ,

(i) Determine whether the function is a polynomial function.

(ii) If the answer is yes to Part (i), find the degree, the number of roots, the leading coefficient, the y-intercept and the end behavior of the function.

(a) $f(x) = x^{3/2} + x + 2.$

(b) $g(x) = x^2 - 2x + 1$

(c) $h(x) = 2x$

(d) $i(x) = 0.5(x-2)(x+3)(x+1)(x+6)$

(e) $j(x) = -\frac{1}{4}(x-7)(x-2)(x+3)(x+1)(x+6)$

(f) $k(x) = |x|$

(g) $l(x) = \frac{x^2+1}{x-1}$

3. An oil slick is expanding as a circle. The radius of the circle is increasing at the rate of 20 meters per day. Express the area of the circle as a function of d, the number of days elapsed.

4. By cutting away identical squares of side length x cm from each corner of a rectangular piece of cardboard and folding up the resulting flaps, an open box may be made. If the cardboard is 18 cm long and 12 cm. wide, find a function that models the **volume** of the box as a function of x.