3.5: Dividing Polynomials

- $\underset{\text{Dividend}}{P(x)} = \underset{\text{Divisor}}{d(x)} \cdot \underset{\text{Quotient}}{q(x)} + \underset{\text{Remainder}}{r(x)}$ $d(x) \neq 0$ and $r(x)$ is zero or degree of $r(x)$ less than the degree of $d(x)$.
- In rational form: $\dfrac{p(x)}{d(x)} = q(x) + \dfrac{r(x)}{d(x)}$
- The Division Algorithm states that, given a polynomial dividend $p(x)$ and a non-zero polynomial divisor $d(x)$ where the degree of $d(x)$ is less than or equal to the degree of $p(x)$, there exist unique polynomials $q(x)$ and $r(x)$ such that

$$p(x) = d(x)q(x) + r(x)$$

  where $q(x)$ is the quotient and $r(x)$ is the remainder. The remainder is either equal to zero or has degree strictly less than $d(x)$.
- If $r(x) = 0$, then both $d(x)$ and $q(x)$ are factors of $p(x)$.

Long Division

- How to do long division:

  Compare the leading terms, find a term for quotient, multiply by divisor; write the result underneath the dividend with like terms underneath each other and subtract the result from the original. Then repeat with the new polynomial resulting from the subtraction as the new dividend. Stop when the polynomial resulting from subtraction has degree less than the quotient.

Synthetic Division

- **Synthetic division** is a shortcut that can be used when the divisor is a binomial in the form $x - k$. In synthetic division, only the coefficients are used in the division process.
- How to: Given two polynomials, use synthetic division to divide.

  Write k for the divisor.

  Write the coefficients of the dividend.

  Bring the lead coefficient down.

  Multiply the lead coefficient by k. Write the product in the next column.

  Add the terms of the second column.

  Multiply the result by k. Write the product in the next column. Repeat steps 5 and 6 for the remaining columns.

  Use the bottom numbers to write the quotient. The number in the last column is the remainder and the remainder has degree 0, the next number from the right is the coefficient of the term of degree 0 (the constant term), the next number from the right is the coefficient of the term of degree 1, then the coefficient of the term of degree 2 and so on.

1. Find the quotient and the remainder of division $x^4 - 2x^3 + x + 3$ by $x^2 + 1$.

2. Use synthetic division to divide the $2x^3 - 7x^2 + 5$ by $x - 4$.

3. Find the quotient and the remainder using long division.
$$\frac{x^3 + 2x^2 + 2x + 1}{x^2 + 2}$$

4. Use synthetic division to divide $P(x) = x^6 - 5x^5 + 8x^4 - 3x^3 - 7x + 6$ by $x - 6$.

5. The volume of a cylinder is changing and its volume is $V(t) = \pi(t^3 + t^2 - 5t + 3)$ at time t minutes. The radius is changing and its radius is $r(t) = (t - 1)$ at time $t \geq 1$ seconds. Find the height of the cylinder. (Hint: The formula for the volume of a cylinder is $V = \pi r^2 h$.)