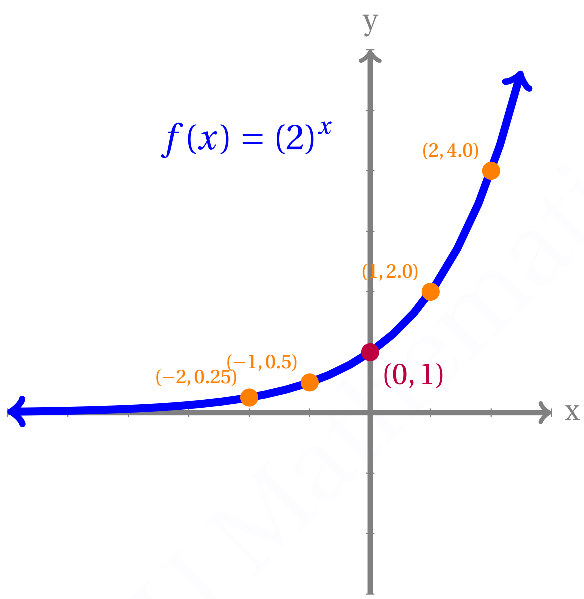
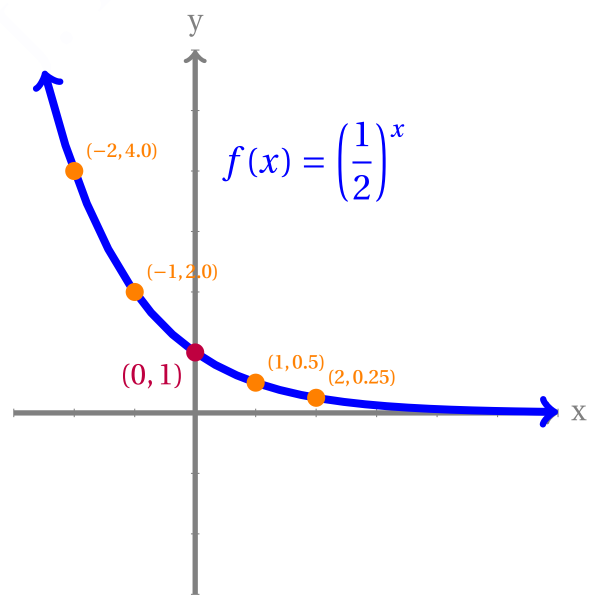
4.1: Exponential Functions

An exponential function is of the form $f(x) = a \cdot b^x$, where

- a is a non-zero real number called the **initial** value and b is any positive real number such that $b \neq 1$.
- The domain of f is all real numbers.
- The range of f is all positive real numbers if $a > 0$.
- The range of f is all negative real numbers if $a < 0$.
- The y-**intercept** is $(0, a)$, and the **horizontal asymptote** is $y = 0$. The graph has NO x-intercept.
- If $b > 1$, then the function is an exponential growth. That is, if $a > 0$, as $x \to \infty$ $y \to \infty$.
- If $0 < b < 1$, then the function is an exponential decay, That is, as $x \to \infty$, $y \to 0$.
- **A few graphs:**





Compound Interest

- $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$

  $A(t) =$ Amount after t years.

  $P =$ Principal

  $r =$ Annual Percentage Rate (APR)

  $n =$ Number of compounding period per year

  $t =$ Number of years

  $\frac{r}{n} =$ Interest rate per period

  $nt =$ Total number of compounding periods

Annual Percentage Yield

- The **annual percentage yield (APY)** of an investment account is a representation of the actual annual interest rate earned on a compounding account. It is based on a compounding period of one year.

$$\text{APY} = \left(1 + \frac{r}{n}\right)^{n} - 1$$

Euler Number e

- The letter e represents the irrational number $\left(1 + \frac{1}{n}\right)^{n} \to e$ as n increases without bound.

  $e \approx 2.718282$ and is the natural base for many real-world exponential models.

Continuous Growth

- We typically use the natural base e for continuous growth. $A(t) = ae^{rt}$.

  $A(t) =$ Amount after time t.

  $a =$ Initial value

  $r =$ Rate of continuous growth

  $t =$ Time elapsed

Continuous Compounding

- $A(t) = Pe^{rt}$

  $A(t) =$ Amount after t years.

  $P =$ Principal

  $r =$ Annual Percentage Rate (APR)

  $t =$ Number of years

1. Identify the exponential functions.

(A) $f(x) = x^{100} + 5x^{50}$

(B) $g(x) = 3(5^{-x})$

(C) $h(y) = 3e^{y-2}$

(D) $i(t) = 0.5(2^{2t-1})$

(E) $j(x) = 0.2^{2x}$

(F) $k(t) = t(t-1)$

2. Evaluate each function at the value given.

(A) $f(x) = e^{-x}$ at $x = 2$

(B) $g(t) = 2^t$ at $t = \pi$

(C) $h(y) = 5(0.5)^y$ at $y = -1$

(D) $i(x) = 9^{-x}$ at $x = 0.5$

3. Chen wants to invest \$3000 at the rate of 6% per year. Help Chen decide; find the amount in the account after 5 years if interest is compounded **(a)** annually, **(b)** semi-annually and **(c)** daily. (Round to nearest dollar.)

4. If \$2000 is invested at an interest rate of 3.5% per year, compounded continuously, find the future value of the investment after the given number of years:

   (A) 2 years.

   (B) 4 years

5. A radioactive substance decays in such a way that the amount of mass remaining after t days is given by the function $m(t) = 13e^{-.015t}$, where $m(t)$ is measured in milligrams.

   (A) Find the mass at time $t = 0$.

   (B) How much of the mass remains after 20 days.

6. A few value of an exponential function $f(x) = ab^x$ is given in the table to the right.

x	f(x)
2	12
3	24
4	48
5	96

   (A) What is $\frac{f(3)}{f(2)}$?

   (B) What is $\frac{f(4)}{f(3)}$?

   (C) What is the value of b?

   (D) What is the value of a?

   (E) Rewrite $f(x)$ using the values in Parts (C) and (D).