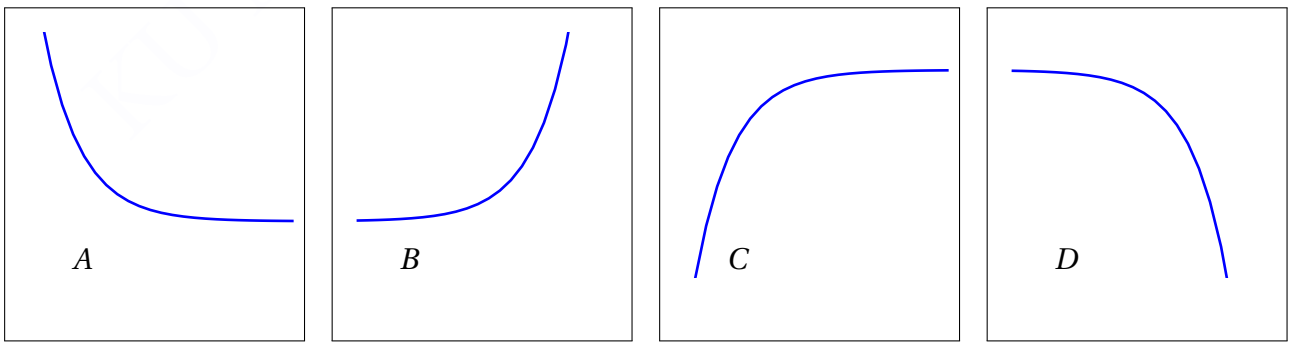
4.2: Graphs of Exponential Functions

- Vertical shift and reflection over x-axis moves the horizontal asymptote.
- The y-intercept is affected by horizontal/vertical shift, vertical stretching/shrinking and reflection over x-axis.
- When an exponential function is transformed, the domain of the resulting function is always $(-\infty, \infty)$ but the range of the resulting function gets affected by vertical shift and reflection over x-axis.
- The horizontal asymptote helps us find the range.
- Reflection over y-axis determines which end behavior is a growth and which one is converging to the asymptote.
- Let $f(x) = ab^{cx+d} + e$. Then the graph of $f(x)$ can be obtained from $g(x) = b^x$ by the following
  - a shift to right/left of $|d|$ units,
  - a horizontal stretching/shrinking of ratio $|c|$.
  - if $c < 0$, a reflection over y-axis.
  - a vertical stretching/shrinking of ratio $|a|$.
  - if $a < 0$, a reflection over x-axis.
  - a vertical shift of $|e|$ units.
- The resulting function's horizontal asymptote is $y = e$.
- The function's range is (e, ∞) if $a > 0$. Its range is $(-\infty, e)$ if $a < 0$.

Another Method of Graphing

- Find if the function is a decay or growth. That is, find the overall shape of the graph:



- Find the horizontal asymptote by finding the shift up or down.
- Find the y-intercept. In a later section, we discuss finding x-intercept.

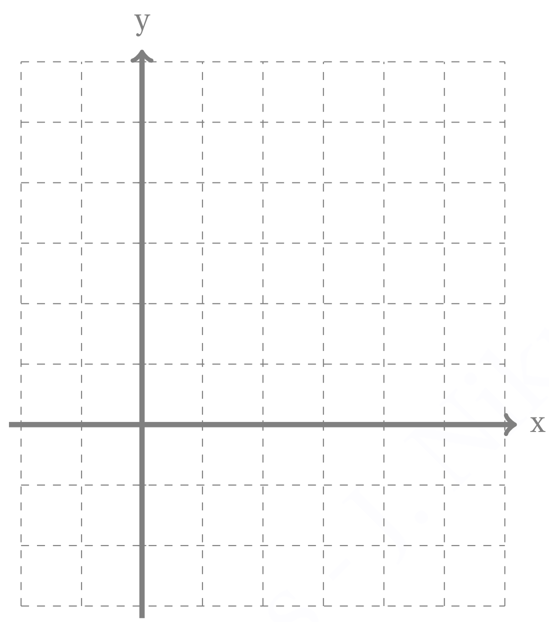
1. Consider the function $f(x) = (0.2)(2)^{-2x+3} - 3$.

   (a) Graph the function.

   (b) What is the horizontal asymptote of the graph?

   (c) What is the y-intercept?

   (d) Find the range of the function.



2. Consider the function $f(x) = -2e^{2x-3} + 1$.

   (a) Graph the function.

   (b) What is the horizontal asymptote of the graph?

   (c) What is the y-intercept?

   (d) Find the range of the function.