7.2: Sum and Difference Identities

- **The two identities:** Sine and Cosine of a sum

  (The interactive proof of the two identities can be found at https://ggbm.at/nf8xydxs
  All the rest are proven using these two formulas and trigonometric identities.)

  $$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
  $$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

- **The derivation of the other sine and cosine identities**

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\underset{\text{Replace } y \text{ by } -y}{\Longrightarrow} \quad \sin(x+(-y)) = \sin(x)\cos(-y) + \cos(x)\sin(-y)$$

$$\underset{\text{Use trig identities for the negative angles}}{\Longrightarrow} \quad \boxed{\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)}$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\underset{\text{Replace } y \text{ by } -y}{\Longrightarrow} \quad \cos(x+(-y)) = \cos(x)\cos(-y) - \sin(x)\sin(-y)$$

$$\underset{\text{Use trig identities for the negative angles}}{\Longrightarrow} \quad \boxed{\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)}$$

- **The entire list to use in your problems in one place:**

  $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ Sine of sum

  $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ Cosine of sum

  $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$ Sine of difference

  $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$ Cosine of difference

- Unknown Angles: To find sine and cosine of an unknown angle using the sum and difference identities, find two well-known angles whose sum or difference equals to the given angle. Then write the proper identity by replacing the x and y with the known angles.

1. Find $\sin(75^\circ) = \sin(30^\circ + 45^\circ)$

2. Find the exact value of $\cos(15^\circ)$.

3. Find the exact value of $\sin(105^\circ)$.

4. Verify the following identity

$$\frac{\sin(x+h)-\sin(x)}{h} = \sin(x)\left(\frac{\cos(h)-1}{h}\right) + \cos(x)\left(\frac{\sin(h)}{h}\right).$$

5. Find $\sin\left(\arcsin\left(\frac{3}{5}\right)+\arccos\left(\frac{12}{13}\right)\right)$.

6. Verify the tangent of sum of angles identity.

$$\tan(\alpha+\beta)=\frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}.$$

Example Video:

- https://mediahub.ku.edu/media/t/1_zph3n20v
- https://mediahub.ku.edu/media/t/1_caahhpte
- https://mediahub.ku.edu/media/t/1_n6tyy64j