7.3: Double-angle/ Half-angle Identities and Reduction Formula

- The Double-angle Identities:

$$\boxed{\sin(2x) = 2\sin(x)\cos(x)}$$

$$\boxed{\cos(2x) = \cos^2(x) - \sin^2(x)} \underset{\text{Using the Pythagorean identity}}{\Longrightarrow} \cos(2x) = 1 - 2\sin^2(x) = 2\cos^2(x) - 1$$

- Power-Reducing Formulas: (Half-Angle Identities:)

$$\sin^2(x) = \frac{1-\cos(2x)}{2} \quad \text{or} \quad \sin^2\left(\frac{\alpha}{2}\right) = \frac{1-\cos(\alpha)}{2}$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2} \quad \text{or} \quad \cos^2\left(\frac{\alpha}{2}\right) = \frac{1+\cos(\alpha)}{2}$$

- Why Power Reduction: In some instances, we need to work with lower powers of trigonometric functions. These identities are very helpful in Calculus.

- Proof of the Double-Angle Identities:

Sine: $\underset{\text{By the sum identity for } x+x}{\Longrightarrow} \sin(x+x) = \sin(x)\cos(x) + \cos(x)\sin(x)$

$\underset{\text{Simplify}}{\Longrightarrow} \sin(2x) = 2\sin(x)\cos(x)$

Cosine: $\underset{\text{By the sum identity for x+x}}{\Longrightarrow} \cos(x+x) = \cos(x)\cos(x) - \sin(x)\sin(x)$

$\underset{\text{Simplify}}{\Longrightarrow} \cos(2x) = \cos^2(x) - \sin^2(x)$

$\underset{\text{Using the Pythagorean identity } \cos^2(x) = 1-\sin^2(x)}{\Longrightarrow} \cos(2x) = 1 - \sin^2(x) - \sin^2(x) = 1 - 2\sin^2(x)$

$\underset{\text{Using the Pythagorean identity } \sin^2(x) = 1-\cos^2(x)}{\Longrightarrow} \cos(2x) = \cos^2(x) - (1 - \cos^2(x)) = 2\cos^2(x) - 1$

- Proof of Power Reducing Formulas

Use $\cos(2x) = 1 - 2\sin^2(x)$ and solve for $\sin^2(x)$ to get $\sin^2(x) = \dfrac{1-\cos(2x)}{2}$

Use $\cos(2x) = 2\cos^2(x) - 1$ and solve for $\cos^2(x)$ to get $\cos^2(x) = \dfrac{1+\cos(2x)}{2}$

1. Use an appropriate Half-Angle Formula to find the exact value of $\sin\left(\frac{3\pi}{8}\right)$.

2. (A) Use an appropriate Half-Angle Formula to find the exact value of $\sin(75^\circ)$.

   (B) Use an appropriate Half-Angle Formula to find the exact value of $\cos(15^\circ)$.

   (C) How are $\sin(75^\circ)$ and $\cos(15^\circ)$ related? Explain why?

3. Combine the two terms in the expression by using a Double-Angle Formula or a Half-Angle Formula.

$$\cos^2(5t) - \sin^2(5t)$$

4. Use the **power reducing formulas**; rewrite the expression in terms of the **first power** of cosine.

(A) $\cos^4(x)$

(B) $\sin^2(x)\cos^2(x)$

5. (A) $\cos(2\arccos(x)) = ?$

(B) Use an appropriate Double-Angle formula and a right triangle; find the exact value of $\sin\left(2\arccos\left(\frac{5}{13}\right)\right)$.

(C) $\tan(2\arcsin(x)) = ?$

6. $\sin\left(\frac{1}{2}\arccos(x)\right) = ?$

7. (Optional: Challenging) Verify the identity using a Double-Angle Formula or a Half-Angle Formula.

$$\frac{2\tan(32^\circ)}{1-\tan(32^\circ)} = \tan(64^\circ) + \sec(64^\circ) - 1$$

Videos:

- https://mediahub.ku.edu/media/t/1_3v2mnxkt
- https://mediahub.ku.edu/media/t/1_caahhpte
- https://mediahub.ku.edu/media/t/1_37ich0ow