Worksheet Nine **MATH 104** Name:_____

WORK ON PROBLEMS IN GROUP OF 2-4. YOUR INSTRUCTOR WILL MARK YOUR GROUP WORK IN CLASS. TURN IN YOUR OWN WORK FOR QUESTIONS MARKED AS "INDIVIDUAL WORK" INDIVIDUALLY; UPLOAD TO CANVAS OR SUBMIT IN CLASS ON THE DUE DATE.

7.5: Solving Trigonometric Equations

- Solving Nonlinear Trigonometric Equation
  - If the inside angles are different, look for identities to make all trigonometric functions in the equation have the same inside angle.
  - If there are different trigonometric functions of the same angle, use identities such as Pythagorean, quotient identities to convert. Or factor such that the same functions are in each factor.
  - Look for a pattern that suggests an algebraic property, such as the difference of squares, difference of cubes, quadratic equation or a factoring opportunity.
  - Substitute the trigonometric expression with a single variable, such as u.
  - Solve the equation the same way an algebraic equation would be solved.
  - Substitute the trigonometric expression back in for the variable in the resulting expressions.
  - Solve for the angle inside in one period. Add multiples of the period if needed (Depends on the domain).
  - Solve for the variable in the inside angle algebraically.
  - **Note: In most cases, we are looking to solve the values for a single trig function but the final goal is to solve for the angle.**
- Finding the Angle:
  - If **Cosine** value $\neq 0$ and $\neq \pm 1$ is found, then the inside angles in one period are either in 1st and 4th quadrant if the value is > 0 or 2nd and 3rd quadrant if the value is < 0.
  - If the **Sine** value $\neq 0$ and $\neq \pm 1$ is found, then the inside angles in one period are either in 1st and 2nd if the value is > 0 or 3rd and 4th if the value is < 0.
  - For every real number x, there is only one angle θ in the period of Tangent which $\tan(\theta) = x$; similarly, for Cotangent function.
  - Secant and Cosecant follow Cosine and Sine respectively.
- Adding a multiple of the period
  - We use integer k to represent an integer multiple of the period.
  - You may test a few values of $k \times$ period by plugging in $k = 0, \pm 1 \pm 2, \ldots$.

1. Solve for all values of x in the following equations.

(a) $\tan(x) - \sqrt{3} = 0$

(b) $\cos(x) = -\frac{\sqrt{2}}{2}$

(c) $\sec(x) + 2 = 0$

(d) $\cos(x) = -1$

(e) $\sin(2x) = 1$

(f) $\tan(3x) + 1 = 0$

2. Find all the solutions, for x, to the following equations.

(a) $\sec^2(x) - 4 = 0$ for x.

(b) $3\tan^2(x) - 1 = 0$ for x.

(c) $3\tan^2(\theta) + 3 = 0$ for θ.

(d) $4\sin^2(3\theta) - 3 = 0$ for θ.

3. (A) Find all solutions to $2\sin(x)\cos(x) - \cos(x) = 0$.

(B) Find all solutions to $(\tan^2(x) - 2)(4\sin^2(x) - 1) = 0$.

4. Find all solutions to the following equations.

(a) $\sin\left(\frac{\theta}{2}\right) = \cos\left(\frac{\theta}{2}\right)$

(b) $\sin(2\theta) = \csc(2\theta)$

5. Find all solutions to $\tan^2(x) - 5\tan(x) + 6 = 0$.

6. Find all solutions to the following equations.

(a) $\tan^2(\theta) + 4\sec(\theta) + 5 = 0$

(b) $2\sin(2\theta) - \cos^2(2\theta) + 1 = 0$

7. Find all solutions to the following equations.

(a) $\sin(2t) = \cos(t)$

(b) $\sin(6t) - \sin(2t) = 0$

INDIVIDUAL WORK

UPLOAD TO CANVAS OR SUBMIT IN CLASS BEFORE DUE DATE. DISCUSSING THESE QUESTIONS IN YOUR GROUP IS ENCOURAGED BUT MAKE SURE YOU ARE TURNING IN YOUR OWN WORK.

8. *(2.5 points)* **This is a question from Section 7.1:** Verify the following trigonometric identity:

$$\cos^2(x) = \frac{\csc(x) - \sin(x)}{\csc(x)}$$

9. Find all solutions for t in $[0, 2\pi)$.

(a) *(2 points)* $\sin(3t) = -\cos(3t)$

(b) *(1 point)* $\cos^2(5t) = 1$

10. *(2.5 points)* Solve $4\sin^2(7t) - 4\sin(7t) + 1 = 0$ for t. (Find all possible solutions.)

Example Video:

- https://mediahub.ku.edu/media/t/1_4bc8kc04