8.6-8.7: Parametric Equations and Graphs

- Parametric Equations Suppose t is a number on an interval I. The set of ordered pairs $(x(t), y(t))$, where $x = f(t)$ and $y = g(t)$, form a plane curve of parameter t. The equations $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ are the parametric equations of the curve.

- Converting Parametric Equations to Cartesian Equations

  - Main goal is to eliminate the parameter between the two equations.
  - One method is to solve for the parameter in both equations and set the results equal to each other. This is not always easy or possible.
  - One method is to find a form of parameter in both equations that can be simplified together. For example, in parametric equations for conic sections, **solve for $\cos(t)$ in one equation and $\sin(t)$ in the other one** and then use the Pythagorean Identity $\sin^2(t) + \cos^2(t) = 1$ to eliminate t.
  - Warning! Some restrictions to domain may be necessary after eliminating the parameter.

  Now, you can complete Problem 1.

- An Application (Projectile Motion)

  A projectile motion problem is usually described with the vertical motion, as a function of time, following the force of gravity, $y(t) = -\dfrac{1}{2} g t^2 + v_0 \sin(\theta_0) t + h_0$ and $x(t) = v_0 \cos(\theta_0) t$ where v_0 is the initial velocity, θ_0 is the angle of projectile, h_0 is the original altitude and g is the gravitational constant.

  Eliminating time will give an equation of vertical versus horizontal distance. That is, the equation of the path that projectile is traveling. Aside from the projectile application, parametrization is used when the graph of an equation is not a graph of a function such as circle and ellipse. Later in Calculus III, we use parametrization to find an equation for a 1-dimensional or a 2-dimensional object residing in the 3-dimensional space.

  Now, you can complete Problem 2.

- Converting to Parametric Equations (Parametrization)

  Set one variable equal to t, e^t, $\cos(t)$ and so on and solve for the other variable.

- Parametrization of Polar Equations: If it is possible to solve for r in a polar equation, that is, the equation can be represented as $r = f(\theta)$, then $\begin{cases} x = f(\theta)\cos(\theta) \\ y = f(\theta)\sin(\theta) \end{cases}$ is a parametrization of the curve.

  Now, you can complete Problem 3-5.

1. Find a Cartesian form of the following parametric equations by eliminating t.

(a) $\begin{cases} x(t) = 5 - 6t \\ y(t) = 5 - t \end{cases}$

(f) $\begin{cases} x(t) = 9\cos(t) \\ y(t) = 9\sin(t) \end{cases}$

(b) $\begin{cases} x(t) = 3 + 4t \\ y(t) = 8 - 2t \end{cases}$

(g) $\begin{cases} x(t) = 4\sin(t) \\ y(t) = 5\cos(t) \end{cases}$

(c) $\begin{cases} x(t) = 5 \\ y(t) = 7t - 16 \end{cases}$

(h) $\begin{cases} x(t) = 2\cos(t) + 1 \\ y(t) = 2\sin(t) \end{cases}$

(d) $\begin{cases} x(t) = e^{2t} \\ y(t) = 17e^{t} \end{cases}$

(i) $\begin{cases} x(t) = 3\cos(t) \\ y(t) = 5\sin(t) - 1 \end{cases}$

(e) $\begin{cases} x(t) = t^2 + 1 \\ y(t) = 15t - 1 \end{cases}$

(j) $\begin{cases} x(t) = 9.8t^2 + 5t \\ y(t) = -2t \end{cases}$

2. An object is thrown in the air with vertical velocity of 8 ft/s and horizontal velocity of 9 ft/s. The object's height can be described by the equation $y(t) = -16t^2 + 8t$, while the object moves horizontally with constant velocity 9 ft/s. Write **parametric equations** for the object's position, and then eliminate time to write height as a function of horizontal position.

3. Parameterize the following equations.

(a) $x^2 + y^2 = 25$

(b) $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$

(c) $\dfrac{(x-7)^2}{16} + \dfrac{y^2}{25} = 1$

(d) $\dfrac{(x-2)^2}{16} + \dfrac{(y+7)^2}{9} = 1$

(e) $25x^2 + 9y^2 = 1$

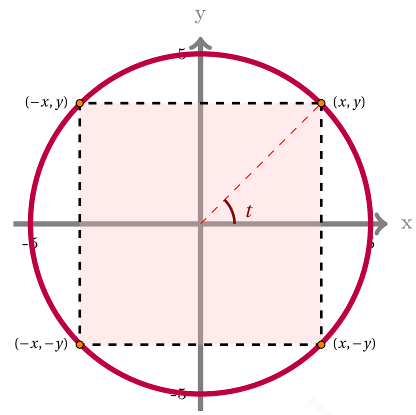
(f) $16x^2 + 9y^2 = 1$

4. Write parametric equations for the line through points $(3, 4)$ and $(6, -3)$.

5. Use the parametric equations of the circle $x^2 + y^2 = 25$ to find the the **maximum area** of a rectangle inscribed in the circle

$$x^2 + y^2 = 25.$$

What are the dimensions of such rectangle?



Standard Equation: $x^2 + y^2 = 25$

Challenging) Find a Cartesian equation of the following parametric equations by eliminating t.

$$\begin{cases} x = 5\left(\dfrac{e^t - e^{-t}}{2}\right) \\ y = 3\left(\dfrac{e^t + e^{-t}}{2}\right) \end{cases}$$

To see some of the domains of parametric curves, refer to the following app: https://www.geogebra.org/classic/ydf4p2n7.

Addtional Example Video:

- https://mediahub.ku.edu/media/t/1_j7e8utyn https://mediahub.ku.edu/media/t/1_kgfn8o90