Worksheet 1: Precalculus

They asked: "How do I study for this course?"
I said: "Learn the material every day."
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In which I replied: "Read the appropriate section of the book before lecture, do the PreLecture assignments, download the lecture notes, follow the lectures without distraction of your phones and other devices, work on worksheets before going to the labs, continue working on your worksheets in the labs to learn the material hands-on, and do the achieve assignments to practice the skills you learned. Make sure to get help in our help room or at one of the free tutoring places on campus if a concept is difficult. Aforementioned are the most important part of learning. Then do all problems on review on your own the week before the exam to study."

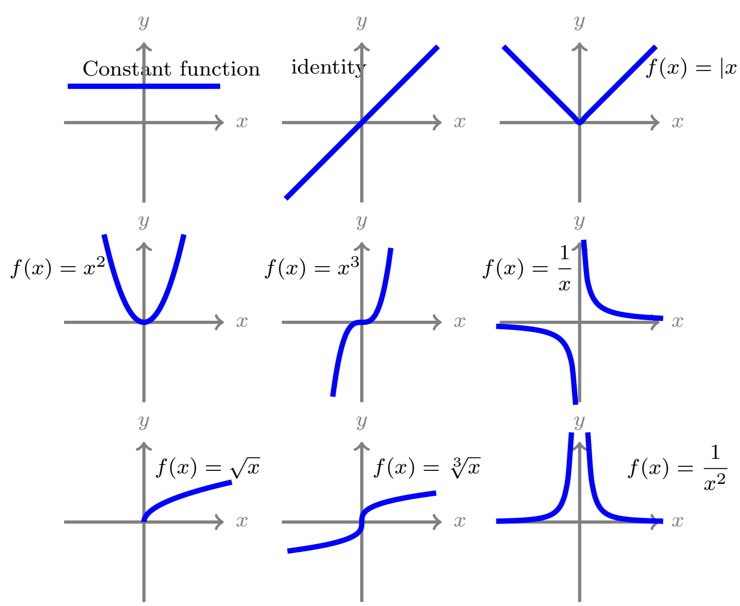
Short Descriptions and Formulas

This week's labs main focus is: ① getting to know each other and the course, ② remembering precalculus, and ③ relearning how to break-down practical problems, and analyzing each piece using common-sense and our precalculus knowledge.

The Absolute Value Function is Piecewise-defined

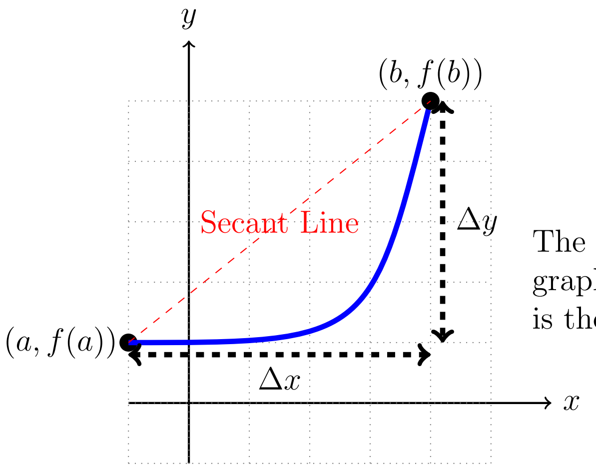
- $|x| = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$ (Check this fact by taking a sample value in each rule.)
- $|x-a| = \begin{cases} x-a & \text{when } x-a \geq 0 \\ -(x-a) & \text{when } x-a < 0 \end{cases} \Longrightarrow |x-a| = \begin{cases} x-a & x \geq a \\ -x+a & x < a \end{cases}$
- https://mediahub.ku.edu/media/t/0_zjd1c0yf

Graphs of Well-known Functions



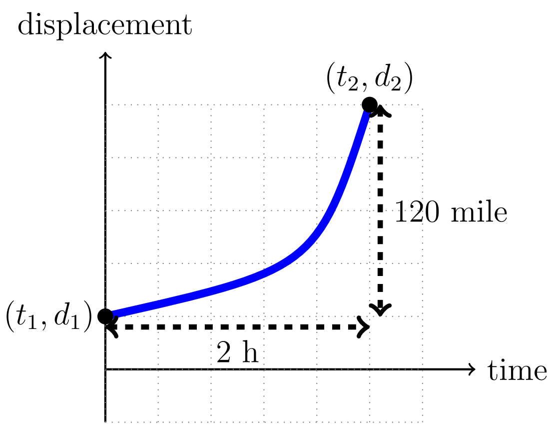
Review of Average Rate

- The average rate of change for function $f(x)$ between $x = a$ and $x = b$ is the ratio of change in y-value to the change in x-value.
  $\dfrac{\text{Change in } y}{\text{Change in } x} = \dfrac{\Delta y}{\Delta x} = \dfrac{f(b) - f(a)}{b - a}$
- The slope of secant line between two points on the graph of f is the average rate of change in f between the two points. The direction of change is the direction of slope.



The average rate of change between two points on the graph (note that y-values come from the function) is the slope of the secant line between the two.

- Average velocity $= \dfrac{\text{displacement between points}}{\text{time elapsed}}$



The average velocity between two points is the ratio of displacement to time.
For example, here the average velocity is
$v_{\text{ave}} = \dfrac{120}{2} = 60 \dfrac{mile}{hr}$

Finding the Average Rate of Change in f over $[a, b]$

(1) Find $f(b)$ and $f(a)$.

(2) Form $f(b) - f(a)$. Simplify and factor if possible.

(3) Form $\dfrac{f(b) - f(a)}{b - a}$, simplify if possible.

(4) Note: If $b = a + h$, then look for a factor h in $f(a + h) - f(a)$.

(5) Try factoring, multiplying by conjugates, trigonometric identities and other methods to simplify h or $b - a$ factor from the numerator and the denominator.

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Ice Breaker Background Story:** This will help you make friends in class and find people to with whom you can study do group works. Take turns to tell us

   (1) Your preferred name,
   (2) What your major is,
   (3) Where you're from,
   (4) Who you are,
   (5) Something we would not know just by looking at you.

2. **Your Math Background and Group Work Requests:** Tell us a bit about your math background and whether you had a math course from us before.

   <u>Write in a piece of paper and turn in to your instructor the answer to the Part (1)-(3).</u>

   (1) What is the highest math class that you have taken?
   (2) Have you taken any math classes at KU?
   (3) Are you feeling comfortable with prerequisite material for this course?
   (4) Are you comfortable working in groups?
   (5) Are there people in class that you prefer to work with?

3. **Background Story:** In this course, we define instantaneous rate of change and to do so, we start we average rate of change. The process of building the concepts requires much algebraic manipulation. This is one of them.

   Question: Find the **average rate of change** of $f(x) = \sqrt{x}$ on the interval $[a, a+h]$. Then simplify as much as possible using conjugates.

4. **Background Story:** We promote understanding different representation of a function.

   Question: Values of the functions f and g are given in the table to the right.

x	$f(x)$	$g(x)$
11	13	13
12	11	14
13	14	12
14	12	11

   (A) $f(g(12)) =$ (B) $g(f(12)) =$

   (C) Average rate of change of f on $[11, 13]$ is

https://mediahub.ku.edu/media/t/0_bkbhbi4q

GroupWork Rubrics:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

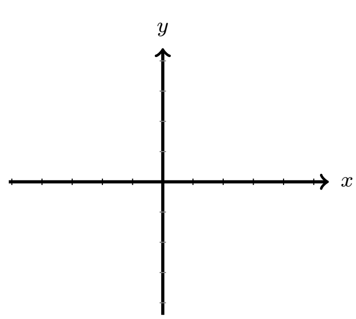
1. **Background Story:** In Chapter 2, we learn about a concept called "limit" of a function as input approaches a value or an infinity. A section of this Chapter is about limits at infinities. The limits at infinities are closely related to asymptotes of the function. That is the limit as input approaches one of the infinities is related to horizontal asymptotes and when the limit tends to infinity as input approaches a value, that limit is related to vertical asymptotes. A review of asymptotes for different functions is a necessity in this course.

   Questions:

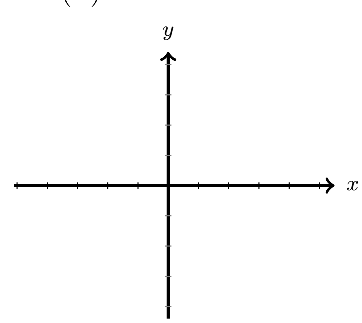
   (A) *(1.25 points)* Find all asymptotes of $f(x) = \dfrac{2x^2 + b}{x^2 - b^2}$, where $b > 0$ is a positive real number (a constant). Label as vertical and horizontal asymptote or say they do not exist. In each case, say if the **input is approaching an infinity** or the **output is approaching an infinity**.

   (B) *(1 point)* Sketch the graph of each function. Your sketch must have the correct overall shape and should accurately show important qualitative features, e.g. x-intercepts, y-intercepts, and asymptotes.[1]

   (i) $y = -e^x + 3$



   (ii) $y = \arctan(x)$



   (C) *(0.25 points)* The function $f(x) = ae^{-bx}$ for $a, b > 0$ is an exponential decay function. $y = 0$ is the horizontal asymptote of f. Explain the significance of horizontal asymptotes in real life exponential decay problems when the input variable is time $t > 0$.

[1] Make sure to draw and mark every asymptote. For VA use: $x =$ and for HA use $y =$.

2. **Background Story:** Absolute value functions are used often in modeling in engineering and probability and statistics. (One example is in rectifying an alternative current.) In your future, you may need to quickly interpret an absolute value function or graph it but for now savor the time and see if it all makes sense.

   Questions: Consider $f(x) = |x - 5| + 3$.

   (A) *(0.25 points)*For what values of x is $y = x - 5$ positive? For what Values of x is $y = x - 5$ negative?

   (B) *(0.5 points)* Rewrite f as a piecewise-defined function. Explain what information did you use from Part (A).

   (C) *(0.25 points)* Graph $y = f(x)$.