Worksheet 13: Sections 3.10 and 4.1

They said: "How do I study for the exam?" I said: "do each problem on the review, write all details, check your work against the solutions, then put the solutions away and redo the questions."

Linear Approximation For x-values near a, the tangent line L_a can be used to approximate the function $f(x)$. That is, if $|b-a|$ is small, then

$$f(b) \approx L_a(b) = f(a) + f'(a)(b-a)$$

Percentage Error

Suppose that we measure some quantity as $x = a \pm \Delta x$ and then want to calculate $f(x)$.

We often want to know how large the error Δf is *relative to the actual value of* $f(a)$.

That is, we want to look at the **percentage error**:

$$\text{Relative error } = \frac{|\text{error of } f|}{|\text{value of } f|} = \frac{|\Delta f|}{|f(a)|}$$

As before, we can use differentials to approximate percentage error:

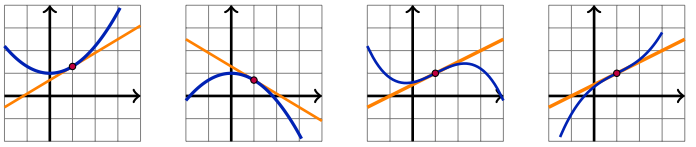
$$\text{Relative error } = \frac{|\Delta f|}{|f(a)|} \approx \frac{|df|}{|f(a)|} = \frac{|f'(a)\,\Delta x|}{|f(a)|}.$$

(In order to convert the relative error to a percentage, multiply by 100%.)

The Effect of Concavity

The accuracy of an approximation using a tangent line is affected by the concavity of the curve. At a point $(a, f(a))$,

concave up	$\Leftrightarrow$	$f''(a) > 0$	$\Leftrightarrow$	graph lies above tangent line	$\Leftrightarrow$	underestimate
concave down	$\Leftrightarrow$	$f''(a) < 0$	$\Leftrightarrow$	graph lies below tangent line	$\Leftrightarrow$	overestimate



Concave Up | Concave Down | Inflection Point | Inflection Point

The greater the absolute value of $f''(a)$, the more the graph diverges from the tangent line.

Summary:

Tangent Line: $\underbrace{y - f(a)}_{dy} = f'(a)\underbrace{(x-a)}_{\Delta x}$

Solve for y: $y = \underbrace{f(a) + f'(a)(x-a)}_{L_a(x)}$

Differentials: $df = f'(a)\Delta x$

Find $f(x)$ and Δx.
df approximates Δf.

Linear Approximation:

$L_a(x) = f(a) + f'(a)(x-a)$

Find $f(x)$ and a.

Group Work Portion of the Worksheet

Names: _____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Disclosure:**

   (i) **What is the point of recitation sessions in Math?** Recitation sessions are where you solve problems with your peers while there is someone in the room who can help you when you are all stuck in a problem.

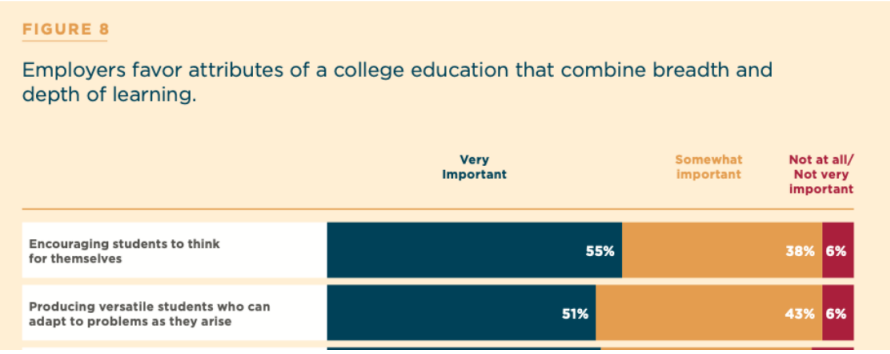
   (ii) **Do I need to prepare before lab and be ready?** Yes, so you can contribute to your team and ask questions while you are there. Otherwise, you are too confused to even be stuck in a problem. Remember attempts, even if they are wrong, are the first step in solving.

   (iii) **How many hours do I need to spend on Math 125?** The minimum recommended hours for any STEM course outside the class, as a rule of thumb by all exports in the fields, is twice the the number of hours spent in the class. Note that this is a minimum and those with weaker background in the subject will need more so they can catch up with the background and learn new material. That is, 12 hours of STEM courses is equal to a full time job, roughly.

   (iv) **How do I know if I don't have a strong background?** If you are having difficulty with algebraic manipulations, you have low trigonometric skills or you have not done many new problems on your own in the past (the last part is really essential).

   (v) **Could I reverse the course of my learning?** YES! Many hours of trying, reasoning on your own and may be talking it out at load with friend(s), classmate(s) or instructor(s) can reverse the course of your learning. Remember the most important part of learning is being able to look at a new problem within the content of the section you are studying and attempt at it, if needed multiple times, until it is solved or partially solved. Don't let your mistakes to discourage you.

   (vi) **How do I get help?** Go to lab sections, group tutoring, our help room and our office hours. Bring your problems, work on them and ask for hints when you are stuck. Talk through the material with friends. Most importantly, think for yourself! Learn how to solve problems on your own.



[1]

[1]The AAC&U surveyed nearly 500 executives and hiring managers from businesses of varying size.

2. **Background Story:** Let's talk about related rates again. This worksheet is shorter than usual so you can discuss the problems on Achieve or the related rate problems from the past worksheet, lecture notes or videos. Now that you have a new set of problems in Section 3.10, use the problem solving skills that you learned and solve real life problems. Don't be discouraged by mistakes, your mistakes are the steps into learning. We are rooting for you, you can do it!

   Questions: A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point $(9, 3)$, its x-coordinate decreases at a rate of $4\,\text{cm/sec}$. How fast is the distance from the particle to the origin changing at this instant?

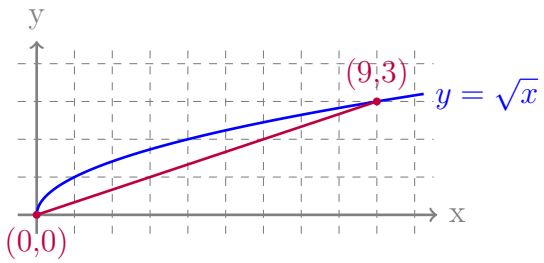
   (i) *Identify and name the quantities in the problem:*

   (ii) *What rate is the problem searching for?*

   (iii) *What are the values of the known variables and rates?*

   (iv) *Which equation(s)* **relates** *the quantities?*

   (v) *Solve.*



3. **Background Story:** Sometimes we can work backward! The expected error in the function is given and you are asked to find the acceptable error in the input measurements.

   Formula to use: $\Delta f \simeq df = f'(a)\Delta x$

   Questions: The standard dime is a cylinder with a volume of $340\,mm^3$ and a height of $1.35\,mm$. In an attempt to scam their local candy store, a teenage counterfeiter has determined that fake coins are accepted if they are within $10\,mm^3$ of the volume of a standard dime. The counterfeiter has total control of height when manufacturing coins.

   (a) What are variable(s), constant(s) and errors of interest of interest in this question? How are the variables related? Which of the variable(s) or error(s) are set? Which ones are you computing?

   (b) What radius should the fake coins have? Round answers to 4 decimal points.

   (c) In order to produce acceptable fake coins, what absolute error is acceptable in the coin's radius? what percentage error is acceptable?

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: _____

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

4. **Background Story:** What does it mean to approximate linearly? How do we find the function, the a-value and x?

   Questions:

   (A) *(0.25 points)* Which of the following statements are True?

   (i) Linear approximation means using a linear function to approximate a nonlinear function in a small window of inputs.

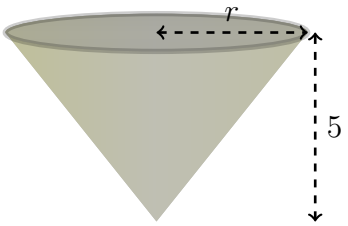
   (ii) Linear approximation means using a quadratic function to approximate a linear function in a small window of inputs.

   (B) *(0.5 points)* What linear function approximate a differentiable function $f(x)$ in a small window of input around $x = a$?

   (C) *(2 points)* Use a **linear approximation** to estimate the value of $\sqrt[3]{29}$. (Round to 4 decimal places.)

   (D) *(0.75 points)* Is this an over-estimate or under-estimate of the actual value?

5. *(3.5 points)* The height of a **conical** vessel is known to be precisely $5\,cm$, and the radius of the base measured as $4\,cm$ with a possible error of $0.1\,cm$. Use **differentials** to estimate the **absolute error** and **percentage error** in computing the **volume** of the vessel.



$$V = \frac{\pi r^2 h}{3}$$