Worksheet 14: Sections 4.2, 4.3 and 4.4

They said: "No one works in a group with me. No one looks like me, sounds like me or has enough patience with me." I am asking: " Be inclusive! This will also enhance your experience with the universe!" Generally, the best way to learn is teach others. SO keep talking and writing math.

Critical Numbers and Fermat's Theorem

A number c in the domain of f is called a critical number if either $f'(c) = 0$ or $f'(c)$ does not exist.

Fermat's Theorem

If f has a local extremum at $x = c$, and $f'(c)$ exists, then $f'(c) = 0$.

That is, if f has a local max or min at c, then c is a critical number of f.

The Extreme Value Theorem

If f is continuous on a **closed** interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

The Closed Interval Method

To find the absolute extreme values of a continuous function f on a closed interval $[a, b]$:

(1) Find the values of f at the **critical numbers** in (a, b).

(2) Find the values of f at the **endpoints** (namely a and b).

(3) Compare the y-values. The largest value is the absolute maximum value; the smallest value is the absolute minimum value.

First Derivative Test for Local Extrema

Suppose that c is a critical number of a continuous function f.

(I) If f' changes from positive to negative at c, then f has a local maximum at c.

(II) If f' changes from negative to positive at c, then f has a local minimum at c.

(III) If f' does not change sign at c, then c is not a local extremum.

Second Derivative Test for Local Extrema

Suppose that f'' is continuous near c and $f'(c) = 0$.

(I) If $f''(c) < 0$, then $(c, f(c))$ is a local maximum.

(II) If $f''(c) > 0$, then $(c, f(c))$ is a local minimum.

(III) If $f''(c) = 0$, then we cannot draw any conclusion.

1. **Background Story:** Absolute value functions have corners when the inside function changes sign. So to find all critical points of a composition of absolute value with an inside function, find all values when the sign of inside function changes and all the critical points of the inside function.

   Questions:

   (A) Solve $x^2 + 6x - 7 > 0$. Solve $x^2 + 6x - 7 < 0$.

   (B) For what values of x, does $f(x) = |x^2 + 6x - 7|$ have corners?

   (C) For what values of f, is $f' = 0$?

   (D) Find the absolute extrema of $f(x) = |x^2 + 6x - 7|$ on the interval $[-8, 0]$.

Optional: Let $f(x)$ be a polynomial. A basic fact of algebra states that c is a root of $f(x)$ if and only if $f(x) = (x - c)g(x)$ for some polynomial $g(x)$. (That is, $x - c$ is a factor of $f(x)$.) We say that c is a multiple root of $f(x)$ if $f(x) = (x - c)^2 h(x)$ where $h(x)$ is a polynomial. (That is, $x - c$ is a factor of multiplicity at least two of $f(x)$.)

(a) Show that c is a multiple root of $f(x)$ if and only if c is a root of both $f(x)$ and $f'(x)$.

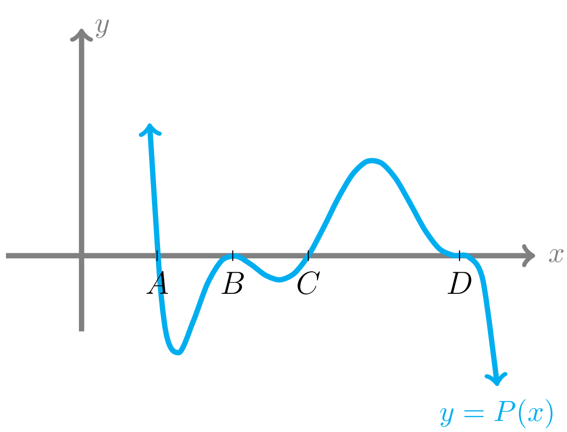
Hints: Verify two statements, "If c is a multiple root of $f(x)$ then c is a root of both $f(x)$ and $f'(x)$" and "If c is a root of both $f(x)$ and $f'(x)$ then c is a multiple root of $f(x)$." I see a product rule in your future in verifying the first statements. Start with what it means for $f(x)$ to have a multiple root the take the derivative and find a factor of $f'(x)$. I also see a product rule in verifying the second statement. This time start with what it means for f to have a root and take the derivative. Then use the fact that c is a root of $f'(x)$ to find a condition on $g(x)$ statement.

(b) Use part (a) to determine whether $c = -1$ is a **multiple** root of each of the following polynomials:

(i) $g(x) = x^5 + 6x^4 + 2x^3 - 20x^2 - 25x - 8$

(ii) $f(x) = x^5 + 6x^4 + 2x^3 - 20x^2 - 27x - 10$

(c) Below is the graph of a polynomial with the roots at A, B, C and D. Which of these are multiple roots? Explain your reasoning using the result of (a).



GroupWork Rubrics:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

2. Let $f(x) = x^{\frac{8}{3}} - 8x^{\frac{5}{3}}$.

   (a) *(3 points)* Identify and classify the **local extrema** of $f(x)$ using the **first derivative test** and **second derivative test.**

(b) *(2 points)* Find the intervals of increasing/ decreasing, concavity and inflection point(s) of the function, if they exist.

(c) *(2 points)* Find the **absolute extrema** of $f(x)$ on the closed interval $[-1, 8]$.