Worksheet 18: Section 5.1

I said: " Integration is arguably the most fundamental concept in Calculus. It will show up in every course where Calculus is a prerequisite. Please keep going to classes and completing your assignments. For us, it has been a semester with post lock-down deficiencies and extreme reductions in resources. Despite it all, watching your progress and a huge improvement in your math knowledge and skills keep us going. Please continue the hard work. Don't slow down! "

Velocity and Distance

The area under the graph of $v(t)$ on a time interval $[a, b]$ measures the net distance traveled, or displacement, between times a and b.

$$\begin{aligned}\text{Units of area under the graph of } v(t) &= \text{units of } t \times \text{units of } v(t) \\ &= \text{time } \times \frac{\text{distance}}{\text{time}} \\ &= \text{distance.}\end{aligned}$$

Summation Notation:

The notation $\sum_{j=m}^{n} a_j$ means $a_m + a_{m+1} + a_{m+2} + \ldots + a_{n-1} + a_n$.

- $\sum$ is the Greek letter Sigma (mnemonic for "sum.")
- The notation $\sum_{j=m}^{n}$ tells us to start at $j = m$ and to end at $j = n$.
- a_j is called the **general term** and j is the **summation index**.

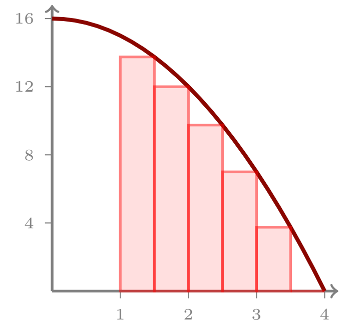
Riemann Sum:

The estimate for the area under the graph of a continuous, positive function $f(x)$ on an interval $[a, b]$ is

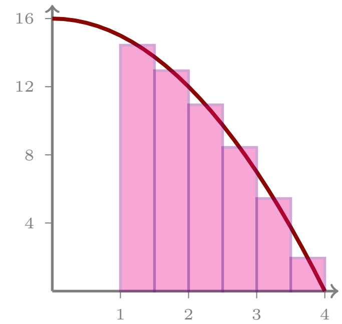
$$\sum_{i=1}^{n} f(x_i)\,\Delta\, x = f(x_1)\,\Delta x + f(x_2)\,\Delta x + \ldots + f(x_n)\,\Delta x$$

and the exact area is

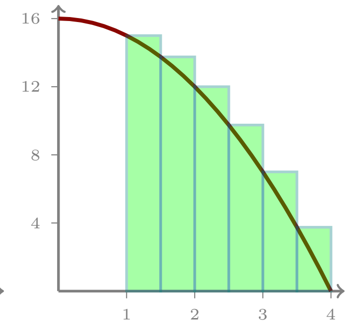
$$\begin{aligned} A &= \lim_{n\to\infty} R_n = \lim_{n\to\infty} \left(f(x_1)\,\Delta x + \ldots + f(x_n)\,\Delta x\right) \\ &= \lim_{n\to\infty} \sum_{j=1}^{n} f(x_j)\,\Delta x. \end{aligned}$$



Right-Hand point approximation: R_n



Mid point approximation: M_n



Left-Hand point approximation: L_n

Approximation of the net area over $[a, b]$ using Riemann sums.

- If n is the number of divisions, the length of sub-intervals is $\Delta x = \dfrac{b-a}{n}$.
- Choose sample point for each sub-interval. The height of each rectangle over the ith sub-interval is $f(x_i)$ and the area is $f(x_i)\Delta x$.
- Sum all the areas.

Newton's Method (Section 4.8)

To approximate a root of $f(x)$, choose an initial value x_0. Generate successive approximations of the root through the equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Sometimes, Newton's Method does not approximate a root.

GroupWork Rubrics day 2:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** Two points to remember: ① Remember the distance traveled in small interval of time, Δs_i, can be approximated by $\Delta s_i \approx v(t_i)\Delta t$. ② The sum of all of these small lengths approximate the total displacement and is a Riemann Sum.

   Questions: A car starts moving and continues with positive acceleration for two seconds. The **velocity** of a car is recorded at half-intervals(in meters per second). Use the left- and the right-endpoint approximation to estimate a lower and an upper bound for the net **distance** traveled during the first 2 seconds.

t(in seconds)	0	0.5	1	1.5	2
$v(t)$(in meters per seconds)	0	2	6	11	20

2. **Background Story:** ① The Riemann sum approximations, for a continuous function $f : [a, b] \to \mathbb{R}$, are categorized by the number of sub-intervals n and the sample points. Some examples are L_n which is the the approximation using n intervals and the **left end point** of each interval as samples points. R_n and M_n are similarly defined for approximations using **right-hand** points and **mid-points**.

   ② The net area $=$ (area under the graph of f and above x-axis) $-$ (area above the graph of f and below the x-axis).

   Questions: For $f(x) = x^2 - 7x + 6$ on the interval $[1, 7]$. Calculate R_4, L_4, and M_4. Sketch the graph of f and the rectangles that make up each approximation.

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3. Understand $\sum$ notation .

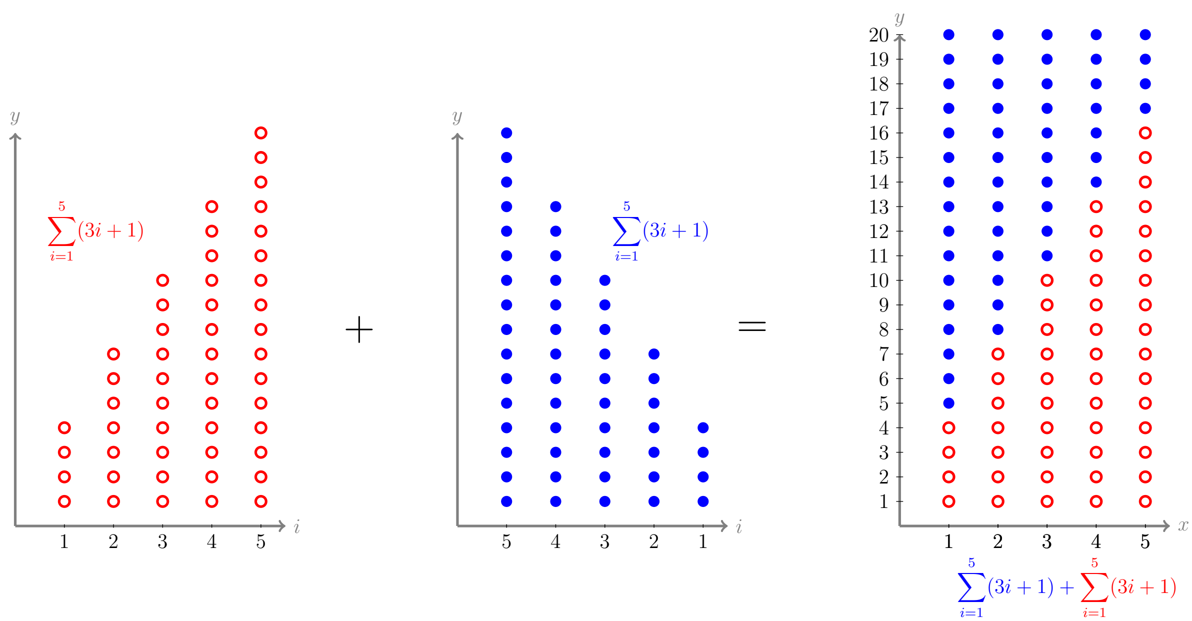
(A) Expand $\sum_{n=1}^{5}(3n+1)$ and evaluate it.

(B) Expand $\sum_{n=0}^{5}(3n+1)$ and evaluate it.

(C) *(Calc 2)* Find the values of c and d such that $\sum_{n=1}^{5}(3n-2)=\sum_{n=c}^{d}(3n+1)$.

(D) *(Computer science)* Find a formula for $\sum_{i=1}^{n}(3i+1)+\sum_{i=1}^{n}(3i+1)$ using the following pattern.

Then find a formula for $\sum_{i=1}^{n}(3i+1)$.



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4. Let $f(x) = x^2 - 8$. Compute the root of this polynomial using Newton's method.

Note that the Newton's formula for the polynomial is $x_{n+1} = x_n - \frac{x_n^2 - 8}{2x_n}$

(A) Start point: $x_0 = 1$.

$x_1 =$
$x_2 =$
$x_3 =$
$x_3 =$
$x_4 =$
$x_5 =$

(B) What is the accuracy after 5 iterations?

(C) Start point: $x_0 = -1$.

$x_1 =$
$x_2 =$
$x_3 =$
$x_3 =$
$x_4 =$
$x_5 =$

(D) Start point: $x_0 = 0.5$.

$x_1 =$
$x_2 =$
$x_3 =$
$x_3 =$
$x_4 =$
$x_5 =$