Worksheet 19: Section 5.2

They were doing optimization problems and they said: "Algebra is difficult. We took it online." I said: " We understand and we are adapting to it but we are expecting everyone catch up in one or two semesters." Then I asked: "Can you tell the consumer that you built this bridge, airplane, data security and $\cdots$ but you were not caught up with what you needed to learn after pandemic?" They laughed but they agreed.

Definite Integral:

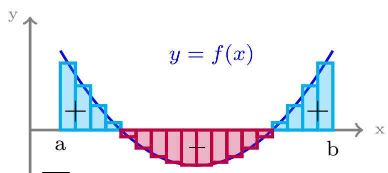
The **definite integral** of f on the interval $[a, b]$ is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

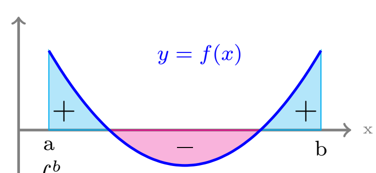
where $\Delta x = \dfrac{b-a}{n}$ and $x_i = a + i\Delta x$, provided that this limit exists.

(i) If f is continuous on $[a, b]$, or if f has only a finite number of jump discontinuities, then f is **integrable** on $[a, b]$.

(ii) The definite integral calculates **net** area. To find the **total** area contained between the graph of a function and the x-axis, calculate $\displaystyle\int_a^b |f(x)|\,dx$.



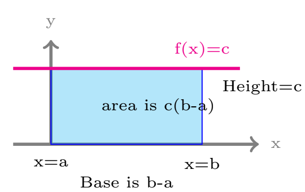
$\sum f(x_i)\Delta x$ is an approximation of the net area.



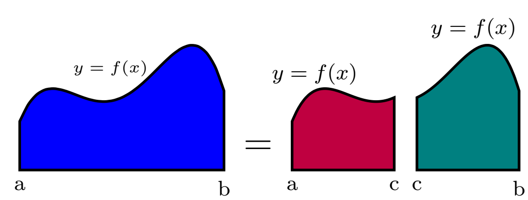
$\int_a^b f(x)\,dx$ is the net area.

Properties of definite integrals:

(i) For a constant c, $\displaystyle\int_a^b c\,dx = c(b-a)$.



(ii) $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$



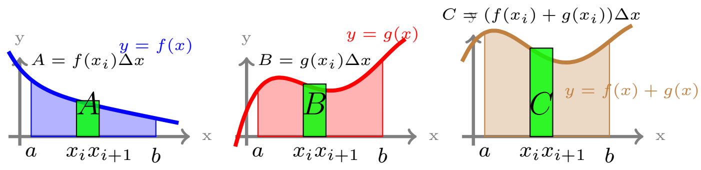
(iii) $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(iv) $\displaystyle\int_a^a f(x)\,dx = 0.$

(v)

$$\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

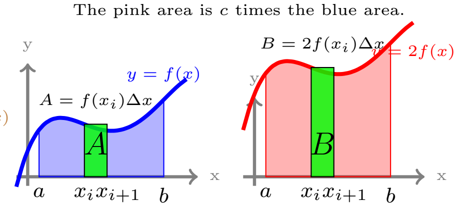
The brown area is the sum of the pink and the blue.



(vi)

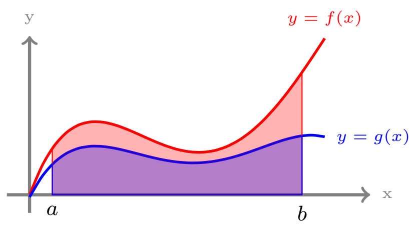
$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$

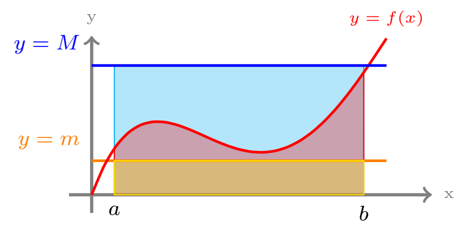
The pink area is c times the blue area.



(vii) If $f(x) \geq g(x)$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$.

(viii) If $m \leq f(x) \leq M$ on the interval $[a, b]$, then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$.





GroupWork Rubrics: Preparedness: ——/1 point, Contribution: ——/1 point, Correct Answers: ——/1 point

Individual Portion of the Worksheet

Name: __________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

1. **Background Story:** Use the properties of definite integrals in the given order.

Part (A):

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

"Distribute": $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

For a constant c $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$

For a constant c, $\int_a^b c\,dx = c(b-a)$

Part (B):

Use once:

Swapping the limits of an integral: $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

Use twice:

Combining the sum of 2 integrals $\int_a^{b} f(x)\,dx + \int_{b}^c f(x)\,dx = \int_a^c f(x)\,dx$

Questions:

(A) *(2 points)* Calculate the definite integral $\displaystyle\int_3^0 \left(6f(x) - \frac{g(x)}{2} + 4\right) dx$ if

$$\int_0^3 f(x)\,dx = 5 \qquad \int_0^3 g(x)\,dx = -3$$

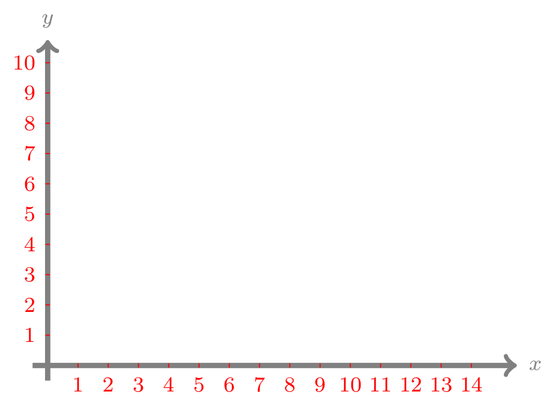
(B) *(1.5 points)* Write $\displaystyle\int_2^{-5} f(x)\,dx + \int_1^{-3} f(x)\,dx - \int_2^{-3} f(x)\,dx$ as a single integral $\displaystyle\int_a^b f(x)\,dx$.

2. **Background Story:** Use the graph and geometry to compute. Remember that the area of a triangle is half of a height times the corresponding base.

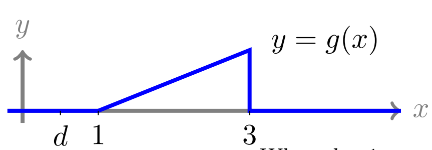
   Questions:

   (A) *(2 points)* Graph $f(x) = |10 - x|$ and calculate the definite integral, using geometry,

$$\int_0^{14} |10 - x| \, dx.$$

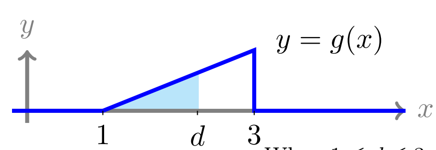


   (B) *(0.75 points)* Consider $g(x) = \begin{cases} (x-1) & \text{When } 1 \le x \le 3 \\ 0 & \text{Otherwise} \end{cases}$. Define $A(d)$ to be the area entrapped between the graph of the function g and x-axis when $0 \le x \le d$ in terms of d.



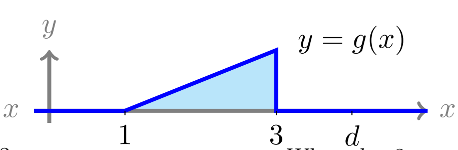
When $d < 1$.

$A(d) =$



When $1 \le d \le 3$.

$A(d) =$



When $d > 3$.

$A(d) =$

   (C) *(0.75 points)* Write $A(x)$, from Part (C), as a piecewise-defined function.