Worksheet 2: Precalculus Review

They asked: "How do I solve problems that I never encountered before?" I answered: "Learn the related material from lecture and lab; read the problem and identify the objective; then identify intermediate objectives and use all your math knowledge, from the beginning of your time to now, logic and common sense to translate each intermediate objective to math and to solve."

Short Descriptions and Formulas

Laws of Logarithms

Let a and b be positive numbers and let x and y be real numbers. Then,

1. $b^x \cdot b^y = b^{x+y}$
2. $\dfrac{b^x}{b^y} = b^{x-y}$
3. $(b^x)^y = b^{xy}$
4. $b^0 = 1$
5. $b^1 = b$
6. $(ab)^x = a^x b^x$
7. $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

Laws of Logarithms

If m and n are positive numbers and $b > 0$, $b \neq 1$, then

1. $\log_b(mn) = \log_b(m) + \log_b(n)$ Log of a product is the sum of logs
2. $\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n)$ log of a quotient is difference of logs
3. $\log_b(m^n) = n\log_b(m)$ log of an exponential is the product of exponent $\times$ log of base of the exponential
4. $\log_b(1) = 0$ log of one is zero
5. $\log_b(b) = 1$ log of the base is one
6. $\log_c(x) = \dfrac{\log_b(x)}{\log_b(c)}$ Change of base formula

Fun Fact:

Numbers 1-6 of laws of logarithm are consequences of laws of exponents.

- **Show "Law of exponent 1 $\implies$ Law of Logarithm 1":**
- To **combine** two or more logs, make sure that the coefficients become exponents inside the log, then use the log of sum and difference law.
- To **expand** a log, use the law of sum and differences first, and then use the law of exponents and simplify using inverse function property.

Group Work Portion of Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** One of the important subjects in Calculus I is optimization which has applications in STEM and business. To be able to solve optimization problems in Calculus I, we will need to simplify expressions and solve equations.

   Questions: Consider the function

   $$f(x) = 2(x+1)^{1/2} + \frac{10x}{(x+1)^{1/2}}.$$

   (A) Simplify and factor the function f as much as possible in form of a quotient function.

   (B) Find the **zeros** of f.

   (C) Find the **domain** of of the function.

   Video: https://mediahub.ku.edu/media/t/0_40cgmxuc

2. **Background Story:** We are interested in relating rates of change in variables when variables are related. The first step to do so is finding the relationships between variables in applied problems The following problem promotes that.

   Questions: Albert lets go of his balloon and the balloon starts rising vertically at a **constant speed** of 6 feet per second. Mom who is standing 18 feet from Albert is looking directly at the balloon, which was originally in her horizontal line of sight, stays frozen in her place and keeps looking straight at the balloon as it is rising. Let y be the vertical distance between Albert and the balloon, in feet, and θ be the angle of mom's line of sight with horizon.

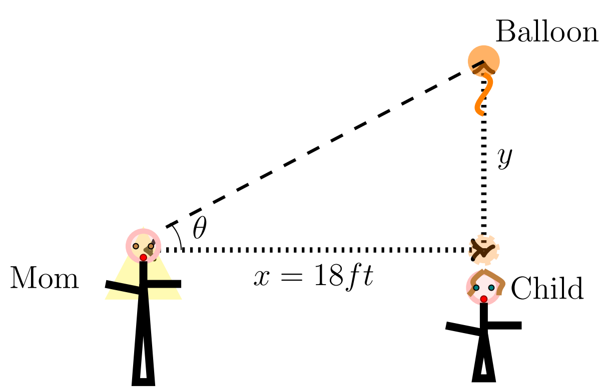
   (a) Express the distance between the mom and the balloon, D, as a function of y.

   (b) Express y as a function of t, time in seconds.

   (c) Express D as a function of t.

   (d) Express θ as a function of y.

   (e) Express θ as a function of t.



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GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

1. **Background Story:** There are instances that we need to find the compositions of two or more piecewise-defined functions. One of these instances is when we need to recognize if a signal is large enough to meet an standard. This problem is not restricted to electrical engineering.

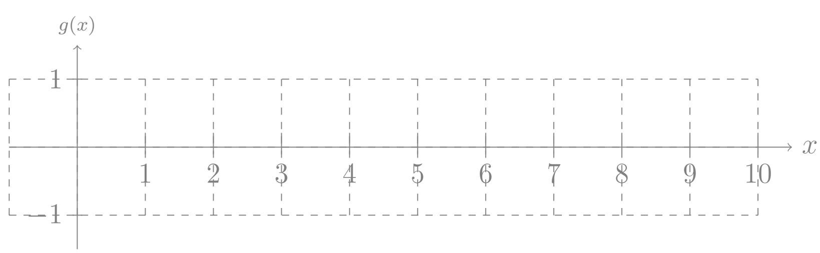
   Questions: Consider $g(x) = \cos(\frac{\pi x}{4})$ restricted to the domain $[0, 8]$.

   (A) *(0.25 points)* For what values of x, $g(x) = \dfrac{1}{2}$?

   (B) *(0.25 points)* For what values of x is $g(x) > \dfrac{1}{2}$?

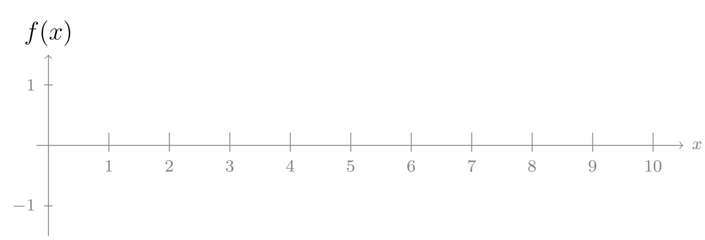
   (C) *(0.25 points)* For what values of x is $g(x) \leq \dfrac{1}{2}$?

   (D) *(0.25 points)* Graph $g(x)$ and mark the part(s) where $y > \dfrac{1}{2}$.



   (E) *(0.25 points)* Use the graph in Part (D) to sketch the graph of $f(x) = \begin{cases} 1 & \text{when } \cos\left(\frac{\pi x}{4}\right) > \dfrac{1}{2} \\ 0 & \text{when } \cos\left(\frac{\pi x}{4}\right) \leq \dfrac{1}{2} \end{cases}$

   restricted to the domain $[0, 8]$.



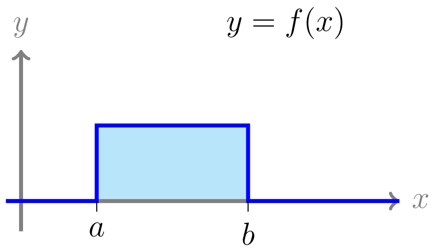
(F) *(0.5 points)* Fill in the blank with interval(s): $f(x) = \begin{cases} 1 & \text{when } x \text{ is in} \quad \_\_\_\_ \\ 0 & \text{when } x \text{ is in} \quad \_\_\_\_ \end{cases}$

(G) *(0.25 points)* Consider the function $h(x) = \begin{cases} 1 & \text{when } x > 1/2 \\ 0 & \text{when } x \leq 1/2 \end{cases}$. How are f, g and h related?

2. **Background Story:** The probability distributions are used in many STEM fields. Most of you are familiar with normal distribution. [1] In earlier physics and engineering courses, you will see uniform probability distribution. A function $f(x) = \begin{cases} k & \text{When } a \leq x \leq b \\ 0 & \text{Otherwise} \end{cases}$ is a **uniform probability distribution function** if the **area** entrapped between the graph of the function, x-axis, $x = a$ and $x = b$ is **one**. The following problem will build skills that you need to understand the distribution and also enforces the concept of functions.

Questions: Consider $f(x) = \begin{cases} k & \text{When } a \leq x \leq b \\ 0 & \text{Otherwise} \end{cases}$.

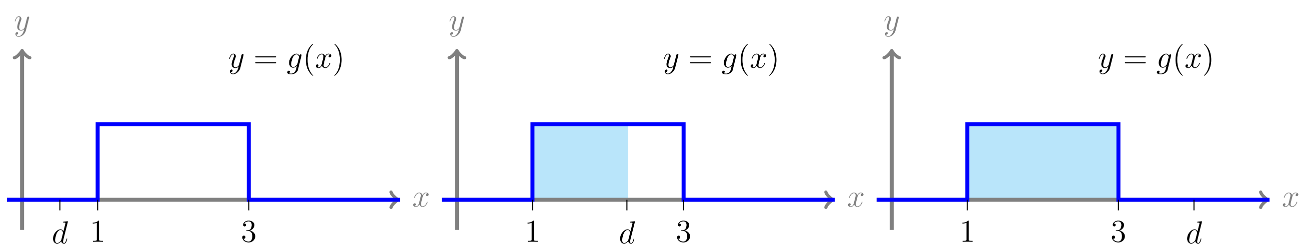
(A) *(0.25 points)* What is the area between entrapped between the graph of the function, x axis, $x = a$ and $x = b$ in terms of a, b and k? (The shaded area in the picture.)



(B) *(0.25 points)* For what value of k is $f(x)$ a probability distribution function?

(Give a value in terms of a and b.)

(C) *(0.75 points)* Consider $g(x) = \begin{cases} 0.5 & \text{When } 1 \leq x \leq 3 \\ 0 & \text{Otherwise} \end{cases}$. Define $A(d)$ to be the area between entrapped between the graph of the function g, x axis, $x = 1$ and $x = d$ in term of d. What is $A(d)$ if $1 \leq d \leq 3$? What is $A(d)$ if $d < 1$? What is $A(d)$ if $d > 3$?



When $x < 1$.　　When $1 \leq d \leq 3$.　　When $d > 3$.

(D) *(0.25 points)* Replace d with x in your answers for Part (C) and write $A(x)$ as a piece wise function.

[1] We discuss a few fact about normal distribution in each of the calculus sequence.