Worksheet 20: Section 5.3

I said: "The review material is posted. Start on them please!"

Antiderivatives

An **antiderivative** of a function $f(x)$ is a function $F(x)$ such that

$$F'(x) = f(x).$$

Calculating an antiderivative involves reversing the derivative process.

Theorem: If $g'(x) = f'(x)$, then $g(x) = f(x) + C$, where C is some constant.

Theorem: If F and G are both antiderivatives of $f(x)$, then $G(x) = F(x) + C$, where C is some constant.

General Antiderivatives

In other words, a function $f(x)$ *has many different antiderivatives*, all of which differ by a constant.

$F(x) + C$ is called the **general antiderivative** of f.

For example, if an object's acceleration is $a(t)$, then

- the object's velocity is described by the general antiderivative $v(t) = A(t) + C$,
- the object's displacement is described by $s(t) = V(t) + Ct + D$, the general antiderivative of $A(t) + C$.

Many formulas for antidifferentiation come from reversing the differentiation formulas that we we already know.

Some Basic Antiderivative

Function	x^n (where $n \neq -1$)	x^{-1}	e^x	a^x
Antiderivative	$\frac{x^{n+1}}{n+1} + C$	$\ln\|x\| + C$	$e^x + C$	$\frac{a^x}{\ln(a)} + C$

Some Basic Trigonometric Antiderivative

Function	$\cos(x)$	$\sin(x)$	$\sec^2(x)$	$\sec(x)\tan(x)$	$\frac{1}{x^2+1}$	$\frac{1}{\sqrt{1-x^2}}$
Antiderivative	$\sin(x) + C$	$-\cos(x) + C$	$\tan(x) + C$	$\sec(x) + C$	$\arctan(x) + C$	$\arcsin(x) + C$

However, this leaves many functions that we do not yet know how to antidifferentiate.

The notation:

$$\int f(x)\,dx = F(x) + C \text{ means that } F'(x) = f(x).$$

We say that $F(x) + C$ is the general antiderivative or the **indefinite integral** of $f(x)$.

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** Use the properties of antiderivatives and the reversing the derivative process to find the indefinite integrals.

   Questions:

$F(x)$	$f(x) = F'(x)$	$f(x)$	$F(x) = \int f(x)dx$
$A(x) = \sec(x) + \tan(x)$	$\frac{dA(x)}{dx} =$	$a(x) = \sec(x)(\tan(x) + \sec(x))$	$\int a(x)dx =$
$B(x) = e^{3x}$	$\frac{dB(x)}{dx} =$	$b(x) = e^{3x}$	$\int b(x)dx =$
$C(x) = 5e^{x}$	$\frac{dC(x)}{dx} =$	$c(x) = 5e^{x}$	$\int c(x)dx =$
$D(x) = \cos(7x)$	$\frac{dD(x)}{dx} =$	$d(x) = \sin(7x)$	$\int d(x)dx =$
$E(x) = \arctan(5x)$	$\frac{dE(x)}{dx} =$	$e(x) = \frac{1}{1 + (5x)^2}$	$\int e(x)dx =$
$F(x) = 5\ln\lvert x\rvert + 7x^{2/5}$	$\frac{dF(x)}{dx} =$	$f(x) = 5x^{-1} + x^{-3/5}$	$\int f(x)dx =$
$G(x) = \tan(5x)$	$\frac{dG(x)}{dx} =$	$g(x) = \sec^2(5x)$	$\int g(x)dx =$
$H(x) = e^{x^2+1}$	$\frac{dH(x)}{dx} =$	$h(x) = xe^{x^2+1}$	$\int h(x)dx =$
$I(x) = \sin\left(\frac{x}{7}\right)$	$\frac{dI(x)}{dx} =$	$i(x) = \cos\left(\frac{x}{7}\right)$	$\int i(x)dx =$
$J(x) = f(x^2 + 5x)$	$\frac{dJ(x)}{dx} =$	$j(x) = (2x + 5)f'(x^2 + 5x)$	$\int j(x)dx =$
		$k(x) = (x + 2)(x - 5)$	$\int k(x)dx =$
		$L(x) = \frac{x^2 + x + 1}{x}$	$\int l(x)dx =$

2. **Background Story:** This was released later. Feel free to complete on the second lab day. Remember velocity is an antiderivative of acceleration and the position is antiderivative of velocity. Every time you take the antiderivative, you need to use initial value to solve for the constant of integration. Follow the following steps to find the **initial velocity** of the car, in feet per seconds, when the brakes were first applied, denoted by v_0, and the **time**, in seconds, that it **takes for the car to stop**, denoted by T_{final}.

   Questions:

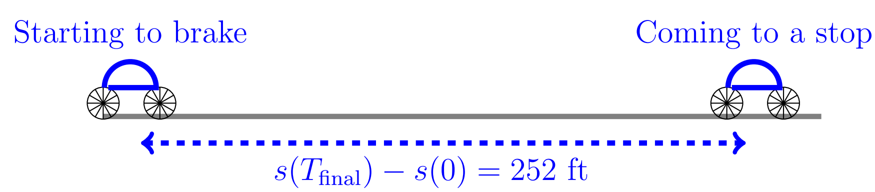
   A car braked with a constant deceleration of 14 ft/s^2, producing skid marks measuring 252 feet before coming to a stop.

   (A) Find an equation for velocity in feet per second, $v(t)$. (You can use the next steps to find the constant of integral, the initial velocity, v_0. For now, keep the initial velocity as a parameter.)

   (B) Find an equation for position in feet, $s(t)$. (You can assume that the brakes are applied at the origin. This means that the initial position is $s(0) = 0$. Continue Keeping the initial velocity parameter.)

   (C) T_{final} is the time passed after the car comes to a stop. So $v(T_{\text{final}}) = \boxed{}$ and $s(T_{\text{final}}) = \boxed{}$. (Fill in the blanks.)

   (D) Use the two equations from Part (D) to solve for v_0 and T_{final}. (You need to solve a non-linear system of equations of two variables to solve for parameters.)



GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

3. *(2 points)* Evaluate $\int \left(t^{2/3} + 5t^2 + e^{2t} + 3\right)\, dt$

4. *(2 points)* A hammer is dropped and it falls for **five** seconds before hitting the ground. Determine how far it falls, assuming an acceleration due to gravity only and no wind resistance.