Worksheet 21: Sections 5.4

The Fundamental Theorem of Calculus I (FTC-1):

If f is continuous on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is **any** antiderivative of f, that is, $F' = f$.

FTC-1: Alternative Formulations:

We frequently use the notation $F(x)\Big|_a^b$ to stand for $F(b) - F(a)$, so that FTC-1 can be rewritten as

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b$$

FTC-1 says that definite and indefinite integrals are related as follows:

$$\int_a^b f(x)\,dx = \int f(x)\,dx\,\Big|_a^b$$

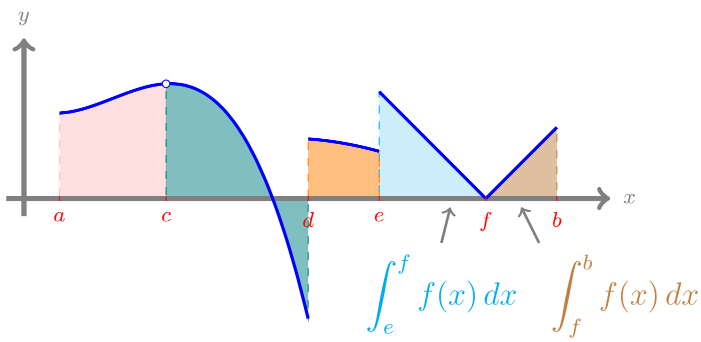
Here is another way of writing FTC-1:

$$\int_a^b F'(x)\,dx \;=\; F(x)\Big|_a^b \;=\; F(b) - F(a).$$

This version of FTC is often referred to as the **Net Change Theorem**, because it says that the integral of $F'(x)$ - that is, the integral of the rate of change of $F(x)$ -is the net change in F.

Finite Numbers of Holes and Jumps:

The statement of FTC-1 applies only to continuous functions, but in fact it can be used to integrate functions whose only discontinuities are a **finite** number of holes or jumps. For example:



$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^d f(x)\,dx + \int_d^e f(x)\,dx + \int_e^f f(x)\,dx + \int_f^b f(x)\,dx$$

and each the five integrals on the right can be evaluated using FTC.

Warning: FTC-1 cannot be used if $f(x)$ has an infinite discontinuity (vertical asymptote). We will explore this further in MATH 126.

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. Use definite integrals, antiderivatives, to compute the definite integrals.

   (A) $\displaystyle\int_0^{\pi/2} \cos(t)\,dt$

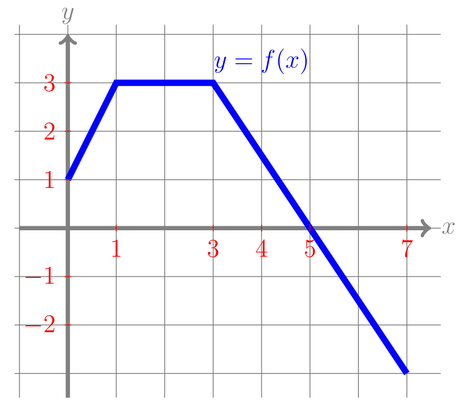
   (B) $\displaystyle\int_1^4 \sqrt{x}(x-2)\,dx$

   (C) $\displaystyle\int_0^1 \frac{1}{x^2+1}dx$

2. **Background Story:** So in lecture, we are going to learn that $\int_c^x f(t)dt$ is an antiderivative of $f(x)$ and can be illustrated as net area. Here we compute two different antiderivatives of $f(x)$. Please note that the difference between two antiderivatives is a constant.

$$A(x) = \int_0^x f(t)\,dt$$

$$B(x) = \int_2^x f(t)\,dt$$



Questions:

(a) Express $A(x)$ as a piecewise function on the domain $[0, 7]$.

$$A(x) = \begin{cases} \boxed{} & \text{when } 0 \le x \le 1 \\ \boxed{} & \text{when } 1 \le x \le 3 \\ \boxed{} & \text{when } 3 \le x \le 5 \\ \boxed{} & \text{when } 5 \le x \le 7 \end{cases} \quad \text{or } A(x) = \begin{cases} \boxed{} & \text{when } 0 \le x \le 1 \\ \boxed{} & \text{when } 1 \le x \le 3 \\ \boxed{} & \text{when } 3 \le x \le 7 \end{cases}$$

(b) Express $B(x)$ as a piecewise function on the domain $[0, 7]$.

$$B(x) = \begin{cases} \boxed{} & \text{when } 0 \le x \le 1 \\ \boxed{} & \text{when } 1 \le x \le 3 \\ \boxed{} & \text{when } 3 \le x \le 5 \\ \boxed{} & \text{when } 5 \le x \le 7 \end{cases} \quad \text{or } B(x) = \begin{cases} \boxed{} & \text{when } 0 \le x \le 1 \\ \boxed{} & \text{when } 1 \le x \le 3 \\ \boxed{} & \text{when } 3 \le x \le 7 \end{cases}$$

(c) Find the minimum and maximum values of A on $[0, 7]$.

(d) Find the minimum and maximum values of B on $[0, 7]$.

3. **Background Story:** When computing integral of absolute value, first rewrite absolute function as a piece-wise-defined function. In the following integral, solve inequality $\cos(x) - \dfrac{1}{2} \geq 0$ to divide the integral into three pieces.

   Questions: Evaluate $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left|\cos(x) - \frac{1}{2}\right| \, dx.$

GroupWork Rubrics:

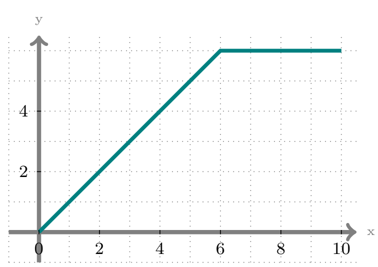
Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2^{nd} lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

4. *(3 points)* The following is a graph of $y = f(x)$. Let $A(x) = \displaystyle\int_0^x f(t)\,dt$ and $B(x) = \displaystyle\int_1^x f(t)\,dt$. Express $A(t)$ and $B(t)$ as piecewise-defined functions on $[0, 10]$.



$$A(x) = \begin{cases} \boxed{} & \text{when } 0 \le x \le 6 \\ \boxed{} & \text{when } 6 \le x \le 10 \end{cases}$$

$$B(x) = \begin{cases} \boxed{} & \text{when } 0 \le x \le 6 \\ \boxed{} & \text{when } 6 \le x \le 10 \end{cases}$$

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