Worksheet 22: Sections 5.4, 5.5 and 5.7

They:"Hang in there! This is the last time before the exam. We are trying our last attempts and we need you to stay with us. After that it is all on you. That said, we are trying the review on Friday, the stop day and holding a few help room hours before the exam; take advantage of those opportunities and ask questions. You have changed a lot, the study habits, the problem solving skills and the general knowledge of the course have improved drastically. But it is important to study close to the exam. Also, did I say we are dropping the lowest scores for 3 weeks of worksheets?"

Cumulative Area Function:

Graph of $f(t) = \frac{d}{dx}A_f(x)$	**Area function** $A_f(x) = \int_a^x f(t)\,dt$
Above the x-axis	Increasing
Below the x-axis	Decreasing
Zero	Local extremum
Increasing	Concave up
Decreasing	Concave down

The Fundamental Theorem of Calculus II (FTC-2):

Suppose f is continuous on the interval $[a,b]$. Then, for all x in $[a,b]$:

$$\frac{d}{dx}\left(A_f(x)\right) = \frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x).$$

$$\frac{d}{dx}\Big(A_f(g(x)) - A_f(h(x))\Big) = f(g(x))g'(x) - f(h(x))h'(x), \text{ where } f,\ g \text{ and } h \text{ are continuous.}$$

Substitution for Indefinite Integrals:

If $u = g(x)$ and $du = g'(x)\,dx$ then we can rewrite this equation as

$$\int f'(g(x))\,g'(x)\,dx \;=\; \int f'(u)\,du \;=\; f(u) + C \;=\; f(g(x)) + C.$$

The Substitution Method for Definite Integrals

If $g'(x)$ is continuous on $[a,b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** From Review material!

   Calculate $\dfrac{d}{dx}\left(\displaystyle\int_{-x^2}^{\sqrt{x}} \tan^5(t)\,dt\right)$ on the domain $0 \leq x \leq \sqrt{\dfrac{\pi}{3}}$. Be sure to quote any theorem used in completing the calculation. **video:** https://mediahub.ku.edu/media/t/1_us3hg8nc

2. **Background Story:** From the review again! Make sure to know the relationship between A_f, f and f'. Use first and second derivative of A_f to find the shape.

   Sketch the graph of an increasing function f such that both $f'(x)$ and $A_f(x) = \displaystyle\int_0^x f(t)\,dt$ are decreasing.

3. **Background Story:** From the review again! You did similar question when reversing the derivative, now use u-substitution for definite integrals.

   $\displaystyle\int_0^9 f(x)\,dx = 4$ show that $\displaystyle\int_0^3 xf(x^2)\,dx = 2$

4. **Background Story:** Practice some of the review questions! These are the set of definite and indefinite integrals in the review that need method of substitution. Please for each integral, specify $u =$ _____, $du =$ _____, the new integral bounds if they apply and the transformed integral. The solve.

Questions: Evaluate the following definite and indefinite integrals:

(A) $\int (5x-4)^8 \, dx$

(B) $\int \frac{dt}{\sqrt{t-5}}$

(C) $\int x^2\sqrt{x^3+1} \, dx$

(D) $\int x^5\sqrt{x^3+1} \, dx$

(E) $\int \cot(x) \, dx$

(F) $\int \sin^7(x)\cos(x) \, dx$

(G) $\int \sec^2(x) e^{\tan(x)} \, dx$

(H) $\int_4^9 (x-8)^{-\frac{2}{3}} \, dx$

(I) $\int_9^{15} (x-8)^{-\frac{2}{3}} \, dx$

(J) $\int_1^e \frac{\ln(x)}{x} \, dx$

(K) $\int_0^{\frac{\pi}{2}} \cos^3(x)\sin(x) \, dx$

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: ________________

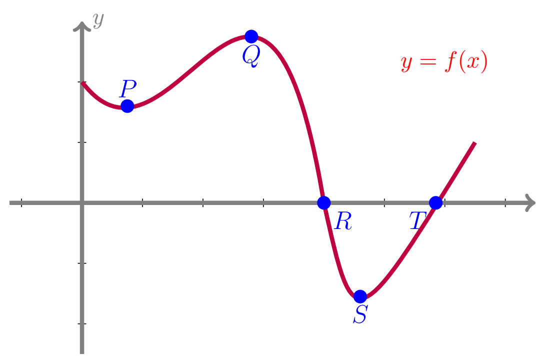
Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

5. **Background Story:** By FTC 2, $f(x) = \dfrac{d}{dx}A_f(x)$.

Questions: Let $A_f(x) = \displaystyle\int_0^x f(t)\,dt$ where $y = f(x)$ is graphed below.



(A) *(0.75 points)* Does A_f have a local minimum at S? Explain your answer.

(B) *(0.75 points)* Where does A_f have a local minimum? Explain your answer.

(C) *(0.75 points)* Where does A_f have a local maximum? Explain your answer.

(D) *(0.75 points)* True or False? $A_f(x) < 0$ for all x in the interval shown. Explain your answer.

6. *(2 points)* Evaluate the following integrals using u-substitution.

(a) $\displaystyle\int_{-\frac{2}{\sqrt{5}}}^{0} x(1-5x^2)^7\,dx$

(b) *(2 points)* $\displaystyle\int_{0}^{\frac{\pi}{2}} \cos(x)\cos(\sin(x))\,dx$