Worksheet 7: Section 3.3 and left-over 3.2

She said: " I was solving the questions correctly for my groups but every time, they preferred his answers over mine. He was giving the exactly same answers." I said: " I am so sorry!" I asked my daughter: "What did you do when this happened to you?" She said: "I had an ally. He said that I did a better job answering the questions than he did. That reduced the pain." I am asking you: " Be an ally!"

Differentiating a Product of Functions

Suppose that $f(x)$ and $g(x)$ are two differentiable functions.

(I) The product $fg(x)$ is differentiable at $x = a$.

(II) $(fg)'(a) = f(a)g'(a) + g(a)f'(a)$, or equivalently

$$\frac{d}{dx}(fg(x))\bigg|_{x=a} = \left(f(x)\frac{dg}{dx} + g(x)\frac{df}{dx}\right)\bigg|_{x=a}$$

(These last two equations say the same thing in Lagrange and Leibniz notation respectively.)

Differentiating a Quotient of Functions

If $f(x)$ and $g(x)$ are differentiable at $x = a$ and $g(a) \neq 0$, then $\left(\frac{f}{g}\right)(x)$ is differentiable at $x = a$ and

$$\left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

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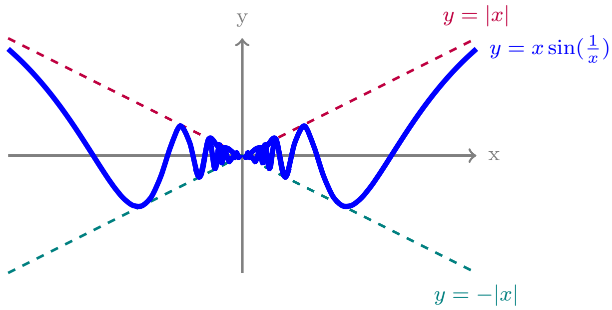
Group Work Portion of the Worksheet

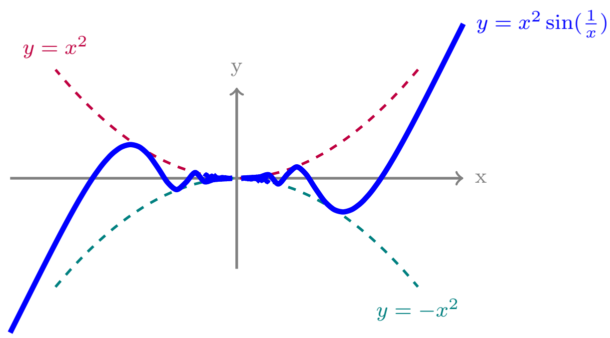
Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** In class, we learned that $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is continuous, by the Squeeze Theorem, but not differentiable at $x = 0$.



In this work, we consider the function $g(x) = \begin{cases} x^2\sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.



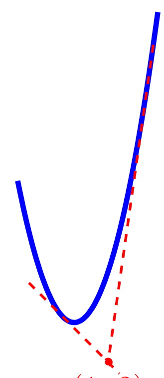
(a) Form the quotient $\dfrac{g(x) - g(0)}{x - 0}$ for $x \neq 0$ and simplify.

(b) Find the $\lim_{x\to 0} \dfrac{g(x) - g(0)}{x - 0}$. (Hint: You will need to use the Squeeze Theorem. Is this limit "the limit definition of derivative"? At what point?)

(c) Is g differentiable at $x = 0$? If yes, what is $g'(0)$?

Click https://ggbm.at/quzevdeg for a visual explanation.

2. **Background Story:** We want to find both points on the graph $f(x) = x^2 + x$ whose tangent line passes through the point $(1, -2)$. Follow the steps.



Questions: Since both points are on graph of the function, $(a, f(a))$ be either of those points.

(A) Find the slope of tangent line to graph of the function $f(x) = x^2 + x$ at $x = a$ using the derivative of the function.

(B) What is the $f(a)$ in terms of a?

(C) Find the slope of tangent line to graph of the function $f(x) = x^2 + x$ at $x = a$ using the two points $(a, f(a))$ and $(1, -2)$.

(D) Find both points on the graph $f(x) = x^2 + x$ whose tangent line passes through the point $(1, -2)$ by setting the two slopes from (B) and (C) equal to each other, then solving for a.

3. **Background Story:** Practice product and quotient rules.

   Questions: Let f and g be two differentiable functions such that

$f(2) = 2$	$f'(2) = -1$	$f(5) = 11$	$f'(5) = -5$
$g(2) = 5$	$g'(2) = -6$	$g(5) = 3$	$g'(5) = 2$

   Evaluate the following derivatives or say it can not be computed.

   (i) $\dfrac{d}{dx}\left(15f(x) - (x^2+1)g(x)\right)\Bigg|_{x=5}$

   (ii) $\dfrac{d}{dx}\left(\dfrac{x^2+f(x)}{g(x)-15}\right)\Bigg|_{x=2}.$

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2^{nd} lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

4. Compute the following derivatives.

   (a) *(0.5 points)* $\frac{d}{dx}\left(5x^6 - 8x^4 + 5x^3\right) =$

   (b) *(0.5 points)* $\frac{d}{dy}\left(5y^6 - 8e^y\right) =$

   (c) *(0.75 points)* $\frac{d}{d\theta}\left(5\theta^6 e^\theta\right) =$

   (d) *(0.75 points)* $\frac{d}{dx}\left(\left(5x^6 - 8x^4 + 5x^3\right)\left(e^x + 11\right)\right) =$

   (e) *(1 point)* $\frac{d}{dx}\left(\frac{5x^6 - 8x^4 + 5x^3}{32e^x + 15}\right) =$