Worksheet 8: Section 3.7

They said: "Why do we have assignment in the week of the exam?" I said: "According to the university policy, we can not cancel any classes. This worksheet is made very short so you can complete both the group work and the individual portion in class. The chain rule is very important for gateway exams."

I added: "Next week, we will have the paper gateway exam in the second lab of the week. Gateway exams cover the entire derivative rules. A good source for review material can be found on Canvas Syllabus module. They are pass and no pass exams but you will have many chances to take them on computer during the week 8 and 9. Students usually prefer paper exams to computer exams so be over prepared to pass them next week."

The Chain Rule

If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $H = f \circ g$ defined by $H(x) = f(g(x))$ is differentiable at x, and its derivative is

$$H'(x) = f'(g(x))\, g'(x) \ .$$

$$\underbrace{f'}_{\text{Outside Derivative}} \quad \underbrace{(g(x))}_{\text{(Inside Untouched)}} \quad \times \quad \underbrace{g'(x)}_{\text{Inside Derivative}}$$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx} \ .$$

Memorize the following trig derivatives for the gateway exams:

$$\frac{d}{dx}\big(\sin(x)\big) = \cos(x) \qquad \frac{d}{dx}\big(\cos(x)\big) = -\sin(x)$$

$$\frac{d}{dx}\big(\tan(x)\big) = \sec^2(x) \qquad \frac{d}{dx}\big(\cot(x)\big) = -\csc^2(x)$$

$$\frac{d}{dx}\big(\sec(x)\big) = \sec(x)\tan(x) \qquad \frac{d}{dx}\big(\csc(x)\big) = -\csc(x)\cot(x)$$

Group Work Portion of the Worksheet

Names: _____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** While you compute the derivative of a composite functions, there may be multiple layers that you have to mind or you may need to use other derivative rules such as sum/difference rule, product rule and quotient rule. Discuss what rule is being used and how you use the derivative of inside and outside.

   Questions: Let $f(x)$ and $g(x)$ be differentiable functions such that

   $$f(1) = -10 \qquad f'(1) = -3 \qquad g(1) = 2 \qquad g'(1) = 1$$

   For each of the following, calculate the value (if it can be done) from the information. Discuss with each other what derivative rules you used.

   (i) $\dfrac{d}{dx}\left(f\left(3x - g(x)\right)\right)\Big|_{x=1}$

   (ii) $\dfrac{d}{dx}\left(e^{f(x)g(x)}\right)\Big|_{x=1}$

   (iii) $\dfrac{d}{dx}\left(g\left(f\left(\dfrac{g(x)}{2}\right) + 11\right)\right)\Big|_{x=1}$

   (iv) $\dfrac{d}{dx}\left(\cos\left(2\pi x\right)\right)\Big|_{x=1}$

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: _____

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

2. *(1.5 points)* Evaluate $\dfrac{d}{dx}\big((x^2+3)\sqrt[3]{x^2+2}\,\big)$.

3. *(2 points)* Evaluate $\dfrac{d}{dy}\left(\dfrac{(y-1)^4}{(y^2+2y)^5} \right)$.

4. (i) *(2.5 points)* Use the **limit definition of derivative** to calculate the derivative of the below function at $a = 5$.

$$f(x) = \frac{1}{\sqrt{5x+7}}$$

(ii) *(1 point)* Verify your answer in part (a) by taking the derivative using a chain rule.