Worksheet 9: Sections 3.8 and Gateway

They said:"I am so anxious! What do I do to pass the paper Gateway?" I said:"Print All-Gateway-Questions page. Do everyone of them. Look at the videos to make sure you have done them correctly. Math is only learned by doing."

I added:"In case, you didn't pass the paper gateway, go to computer lab in the Snow hall and retake the exam. Remember if you pass the exam by answering 8 or more questions correctly out of 10 questions on the exam. When you pass the questions, your gateway score is 100%*. This boast your grade immediately if that is what you are interested ."*

Implicit Differentiation

$$\frac{d}{dx}(y) = \frac{dy}{dx} \qquad \frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx} \qquad \frac{d}{dx}(e^y) = e^y\frac{dy}{dx}$$

Implicit Differentiation:

- Take the derivative of both sides.
- Remember if y is an inside function and use $\dfrac{dy}{dx}$

Derivative of Trig Functions

$$\frac{d}{dx}\big(\sin(x)\big) = \cos(x) \qquad \frac{d}{dx}\big(\cos(x)\big) = -\sin(x)$$

$$\frac{d}{dx}\big(\tan(x)\big) = \sec^2(x) \qquad \frac{d}{dx}\big(\cot(x)\big) = -\csc^2(x)$$

$$\frac{d}{dx}\big(\sec(x)\big) = \sec(x)\tan(x) \qquad \frac{d}{dx}\big(\csc(x)\big) = -\csc(x)\cot(x)$$

Group Work Portion of the Worksheet

Names: _____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. Practice Gateway questions.

Power Rule:

(a) $f(x) = \frac{2}{3}x^{\frac{3}{2}} - \left(\sqrt[3]{4}\right)x + \frac{2}{x^2}$

(b) $p(x) = 16x^3 + \frac{17}{\sqrt{x}} - 10x^{3.1416} + \pi^2$

Product Rule or Product Tricks:

(c) $f(x) = (x^2 + 2x + 5)(x^3 + 1)$

(d) $f(x) = x^{-\frac{1}{2}}\left(1 + x^2 + 3x\right)$

(e) $h(w) = \left(w^{-\frac{1}{3}} - 3w^6\right)(4w^2 - 2w + 7)$

(f) $F(x) = \left(3x^2 + \sqrt{7}\,x - \pi^2\right)\left(\frac{x^4}{3} - \frac{x^2}{\sqrt{10}}\right)$

Quotient Rule:

(g) $f(x) = \frac{x^4 - 3x^2 + 2}{x^2 - 2}$

(h) $f(x) = \frac{x^3 - 1}{\sqrt[3]{x}}$

(i) $m(y) = \frac{1 - 4y^2}{6y^2 + 1}$

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2. Practice more gateway questions.

Chain Rule:

(a) $f(x) = \sqrt[3]{x^4 - 7x}$

(b) $u(t) = \dfrac{1}{\sqrt{t^2 + 2t - 1}}$

(c) $h(s) = (1 + \sqrt{s})^{-\frac{1}{2}}$

(d) $f(x) = (x^2 + 1)^{-10}$

(e) $m(u) = \sqrt{1 + \sqrt{u}}$

Combination of Chain/product/Quotient Rules:

(f) $f(x) = \left(1 + (x^2 + 2)^{\frac{1}{2}}\right)^{\frac{1}{3}}$

(g) $h(w) = \left(1 + \sqrt{w^3 + 3}\right)^4$

(h) $g(t) = (t^3 - 1)^4 (1 + t + t^2)^{-4}$

(i) $h(s) = [(s + 2)^3 (2 - s)]^3$

(j) $h(s) = \sqrt{\dfrac{s^2 + s - 2}{s + 2}}$

(k) $f(x) = \dfrac{5 - x}{2(x - 2)^{\frac{5}{2}}}$

(l) $f(x) = \left(\dfrac{x - 3}{x^2 + 7}\right)^4$

(m) $g(u) = \dfrac{2u - 3}{\sqrt{u^2 - 3u + 4}}$

3. Practice more gateway questions.

Exponential Rules:

(a) $f(x) = (x^2 + 3x)\, e^x$

(b) $f(x) = x^2 2^x$

(c) $f(x) = 3^{x^2+1}$

(d) $f(x) = \dfrac{e^{x^2}}{e^{x-1}}$

(e) $f(x) = \dfrac{e^{-x}}{x}$

(f) $f(x) = x^2 e^{-x}$

(g) $f(x) = e^{-\frac{1}{x^2}}$

(h) $f(x) = x^4 + 4^x$

(i) $f(x) = \left(\dfrac{1}{2}\right)^x$

(j) $f(x) = e^{\sqrt{x}+x^2+2}$

(k) $f(x) = \dfrac{1 + e^{2x}}{2 - e^{2x}}$

4. Practice more gateway questions.

More Exponential and Logarithmic Rules:

(a) $f(x) = e^x \ln(x)$

(b) $f(x) = \ln(3xe^x)$

(c) $f(x) = \ln\left(\dfrac{x-1}{x^2+1}\right)$

(d) $f(x) = \ln\left(\dfrac{e^x}{1+e^x}\right)$

(e) $f(x) = x^2 \ln(2x) + x \ln(3x) + 4 \ln(x)$

(f) $f(x) = \ln\left(\dfrac{1}{x}\right) - \dfrac{1}{\ln(x)}$

(g) $f(x) = x \ln(\sqrt{x}) + \ln(x^{-2})$

(h) $f(x) = (\ln(7x))^{\frac{1}{2}}$

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5. Practice more gateway questions.

Trig Rules:

(a) $w(t) = 17 - \dfrac{\cos(t)}{17}$

(b) $h(a) = 3\sin(a) - \cos(a)$

(c) $f(y) = \dfrac{\sin(y)}{y}$

(d) $m(t) = t\tan(t)$

(e) $p(u) = \dfrac{\tan(u)}{1 - \tan(u)}$

(f) $g(v) = (\sin(v) - v\cos(v))^{-17}$

(g) $H(x) = \dfrac{\sin^2(x) + \cos(x)}{x^2 + x}$

(h) $f(t) = \left(1 + \sqrt{\sin(t)}\right)\left(1 - 2\sqrt{\cos(t)}\right)$

(i) $F(y) = \tan(17 + y)$

More Trig Rules:

(j) $h(r) = 4\cos^7(2 - 4r)$

(k) $l(y) = \sin\left(\sqrt{y}\right) + \sqrt{\sin(y)}$

(l) $m(x) = (\cos(1 - x^2))^{\frac{3}{2}}$

(m) $F(t) = 4t^3 - \dfrac{6}{t} + \dfrac{2}{\sin(3t^2 + 1)}$

(n) $h(x) = (x^2 + x - 1)^5 \sin(5x)$

(o) $f(s) = \dfrac{\tan(2s)}{\cos(1 - 2s)}$

(p) $h(y) = \sin(y^2)\sin^2(y)$

(q) $H(x) = \dfrac{\sin\left(\sqrt{3 - x}\right)}{\sqrt{\tan(4 - x)}}$

(r) $l(t) = \left(1 + (2t + 3\tan(4t))^{-\frac{1}{2}}\right)^{\frac{4}{3}}$

(s) $K(x) = \left(1 - \dfrac{\sin(\pi - x)}{\tan(\pi + x)}\right)^{\frac{2}{3}}$

6. Practice more gateway questions.

More Trig Rules With Exponential and Logarithmic Rules:

(a) $F(t) = \sin(\tan(\pi t))$

(b) $G(x) = \tan(\cos(ex))$

(c) $m(b) = \cos\left(\sin\left(\sqrt{2}\, b\right)\right)$

(d) $k(s) = \cos(\pi \sin(1 - s^2))$

(e) $g(t) = \sin^2(t^2 + \tan(t))$

(f) $R(x) = 2\sin\left(\dfrac{1}{\cos(x)}\right)$

(g) $M(x) = \sqrt{\tan(\sin(4x))}$

(h) $F(y) = 1 + \sqrt{\pi + \cos(\sin(ey))}$

(i) $f(x) = \ln(\sin(x))$

(j) $f(x) = e^{3\cos(2x)}$

GroupWork Rubrics:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

Individual Portion of the Worksheet

Name: _____

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

7. Evaluate the **derivatives**:

   (i) *(2 points)* Find $\frac{dz}{dx}$: $z^5 = e^{xz}$

   (ii) *(2 points)* Find $\frac{dy}{dx}$: $\sin(y) = e^{-\sqrt{x}}$

8. **Background Story:** In computing the derivative of a composite of two function at at instance, two rates at that instance is needed.

   Questions: *(3 points)* The volume of a sphere is given in terms of the radius as $V = \dfrac{4\pi r^3}{3}$. The radius is changing over time. At a particular instant, $r = 7$ and $\dfrac{dr}{dt} = -2$. What is $\dfrac{dV}{dt}$ at that instant?