Week 8-Lab 2: Worksheet 10: Sections 2.6, 3.6 and 3.8

They said: "Oh! That spring break is not coming fast enough!" I said: "True! It has been a really busy two weeks. I appreciate your patience and hard work! I even raise your informal quota for minutes spent while stopping by my office to rant it out this week. We are here to support you, ask us for help. Also remember that you can get some rest next week."

Trig Limits:

- Formulas: $\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1$ and $\lim_{\theta \to 0} \frac{1-\cos(\theta)}{\theta} = 0.$ You may need to regroup.
- You may need to multiply by $\frac{n}{n}$ if you see trig functions $\sin(n\theta)$ and $\cos(n\theta)$.
- You may need to multiply by $\frac{\theta}{\theta}$ when the number of factors $\sin(n\theta)$ exceeds the number of factors θ or multiply by conjugate $\frac{1+\cos(n\theta)}{1+\cos(n\theta)}$ to convert $1-\cos(n\theta)$ to $\frac{\sin^2(n\theta)}{1+\cos(n\theta)}$ function.
- You may need to use identities such as

$$\tan(u) = \frac{\sin(u)}{\cos(u)} \qquad \csc(u) = \frac{1}{\sin(u)} \qquad \text{and} \qquad 1-\cos^2(u) = \sin^2(u)$$

Inverse Trig Derivatives:

$$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\arccos(x)) = \frac{-1}{\sqrt{1-x^2}}$$

Arc Function Video: https://youtu.be/V_jNdz8kdzw

Limit Videos: https://youtu.be/-TgwakufP5Y, https://youtu.be/7zn-XpwZlIU

and https://youtu.be/aLpkMHmilrI

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

GroupWork Rubrics day 2:

Preparedness: ——/0.5, Contribution: ——/0.5, Correct Answers: ——/0.5

1. **Background Story:** Use the methods described in the beginning of the worksheet to evaluate the following trig limits. There are multiple videos available in the Sections 2.6 and 3.6 working through similar these limits.

   Questions: Evaluate the **limits**.

   (i) $\lim_{x \to 0} \frac{1 - \cos(3x)}{\sin^2(2x)}$ (ii) $\lim_{x \to 0} \frac{\sin(20x)}{\tan(5x)}$.

Tuesday/ Thursday labs Should skip either #2 or #3 on Tuesday after the break. Please use help room to work on the other one.

2. The following problem will require a use of chain rule along with an arc function derivative rule.

   Questions: Find $\frac{dy}{dx}$: $y = \arcsin\left(e^{-\sqrt{x}}\right)$

3. **Background Story:** This is an implicit differentiation problem involving trig functions.

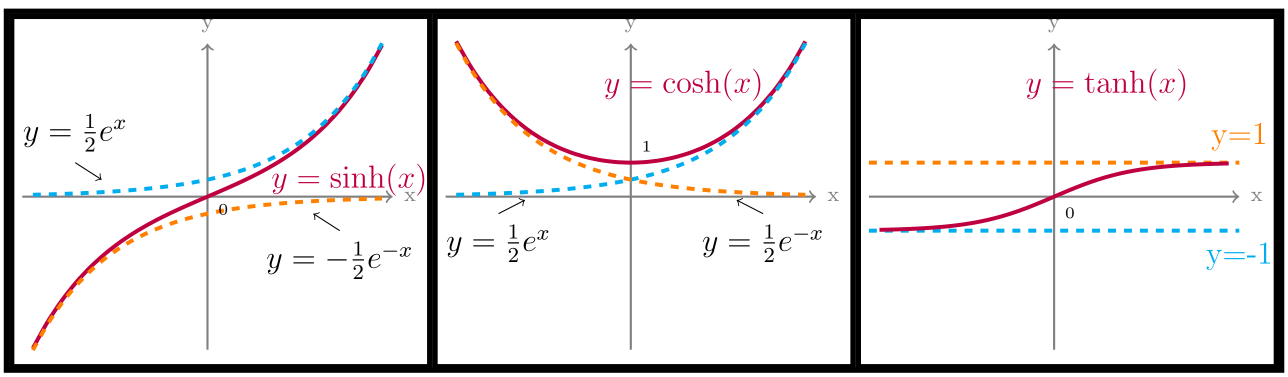
   Questions: Find $\frac{dz}{dx}$: $xz = \arctan(z^2)$

Tuesday/ Thursday labs can skip #4 one on Tuesday. Please use help room to work on it.

4. **Background Story:** Certain even and odd combinations of the exponential functions e^x and e^{-x} arise so frequently in mathematical modeling that they have earned their own special names. In many ways they are analogous to the trigonometric functions, and they have the same relationship to the hyperbola that the trigonometric functions have to the circle. For this reason the are collectively called **hyperbolic functions** and individually called **hyperbolic sine**, **hyperbolic cosine**, **hyperbolic tangent**, etc. The purpose of this problem is to make sense of the relationship between hyperbolic functions and their derivatives.

   Questions: Verify both parts of Item (A) and then choose one or two parts from Item (B) to verify.

$\sinh(x) = \dfrac{e^x - e^{-x}}{2}$	$\cosh(x) = \dfrac{e^x + e^{-x}}{2}$	$\tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$
$\operatorname{csch}(x) = \dfrac{1}{\sinh(x)}$	$\operatorname{sech}(x) = \dfrac{1}{\cosh(x)}$	$\coth(x) = \dfrac{\cosh(x)}{\sinh(x)}$



(A) Verify each of the following hyperbolic identities by replacing the hyperbolic functions on the left of equal sign with the above definition and then simplifying to get to right of equal sign:

(i) $\cosh^2(x) - \sinh^2(x) = 1$

(ii) $1 - \tanh^2(x) = \operatorname{sech}^2(x)$

(B) Verify the hyperbolic derivatives: (Hint: Use Parts (i) and (ii) to Show Part (iii).)

(i) $\dfrac{d}{dx}(\sinh(x)) = \cosh(x)$

(iii) $\dfrac{d}{dx}(\tanh(x)) = \operatorname{sech}^2(x)$

(ii) $\dfrac{d}{dx}(\cosh(x)) = \sinh(x)$

GroupWork Rubrics:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2nd lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

5. **Background Story:** Use the arc derivative rule on the first page and a chain rule to compute.

   Questions: *(2 points)* Evaluate $\frac{d}{dx}\big(\arctan(7x^4)\big)$

6. **Background Story:** One of the reasons we emphasize the trig limits is to strengthen the skills that you need in simplifying expressions. Those skills will come handy in Calculus II. In this specific problem, you may need to convert $1 - \cos(7x)$ using a conjugate, multiply by $\frac{7}{7}$ and simplify.

   Questions: *(2 points)* Evaluate $\lim_{x \to 0} \frac{x \sin(7x)}{(1 - \cos(7x))}$

7. **Background Story:** There may be multiple steps in computing a trig limit of the form $\frac{0}{0}$. Here is an example, Use a trig identity first.

   Questions: *(3 points)* Evaluate $\lim_{x \to 0} \frac{\sin^2(7x)}{\tan^2(4x)}$. (Do not use L'hospital Rule.)