Week 10- Lab 1: Worksheet 13: Sections 3.10 and 4.1

I said: "If you need to pass the Gateway exam, please don't hesitate. Stop by the help room and let me give you feed back. Take it in the lab whenever you can. Whatever you do, do NOT get discouraged! This is part of the learning process.

Linear Approximation For x-values near a, the tangent line L_a can be used to approximate the function $f(x)$. That is, if $|b - a|$ is small, then

$$f(b) \approx L_a(b) = f(a) + f'(a)(b - a)$$

Percentage Error

Suppose that we measure some quantity as $x = a \pm \Delta x$ and then want to calculate $f(x)$.

We often want to know how large the error Δf is *relative to the actual value of* $f(a)$.

That is, we want to look at the **percentage error**:

$$\text{Relative error } = \frac{|\text{error of } f|}{|\text{value of } f|} = \frac{|\Delta f|}{|f(a)|}$$

As before, we can use differentials to approximate percentage error:

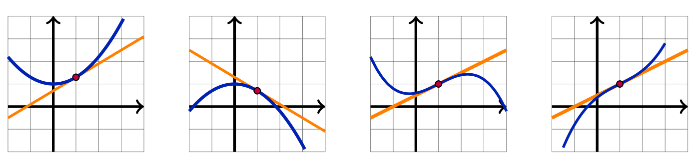
$$\text{Relative error } = \frac{|\Delta f|}{|f(a)|} \approx \frac{|df|}{|f(a)|} = \frac{|f'(a)\,\Delta x|}{|f(a)|}.$$

(In order to convert the relative error to a percentage, multiply by 100%.)

The Effect of Concavity

The accuracy of an approximation using a tangent line is affected by the concavity of the curve. At a point $(a, f(a))$,

concave up	$\Leftrightarrow$	$f''(a) > 0$	$\Leftrightarrow$	graph lies above tangent line	$\Leftrightarrow$	underestimate
concave down	$\Leftrightarrow$	$f''(a) < 0$	$\Leftrightarrow$	graph lies below tangent line	$\Leftrightarrow$	overestimate



Concave Up — Concave Down — Inflection Point — Inflection Point

The greater the absolute value of $f''(a)$, the more the graph diverges from the tangent line.

Summary:

Tangent Line: $\underbrace{y - f(a)}_{dy} = f'(a)\underbrace{(x-a)}_{\Delta x}$

Differentials: $df = f'(a)\Delta x$

Find $f(x)$ and Δx.
df approximates Δf.

Solve for y: $y = \underbrace{f(a) + f'(a)(x-a)}_{L_a(x)}$

Linear Approximation:

$L_a(x) = f(a) + f'(a)(x-a)$

Find $f(x)$ and a.

Group Work Portion of the Worksheet

Names: ____

Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. **Background Story:** Let's talk about related rates again. This worksheet is shorter than usual so you can discuss the problems on Achieve or the related rate problems from the past worksheet, lecture notes or videos. Now that you have a new set of problems in Section 3.10, use the problem solving skills that you learned and solve real life problems. Don't be discouraged by mistakes, your mistakes are the steps into learning. We are rooting for you, you can do it!

   Questions: A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point $(9, 3)$, its x-coordinate decreases at a rate of $4\,\text{cm/sec}$. How fast is the distance from the particle to the origin changing at this instant?

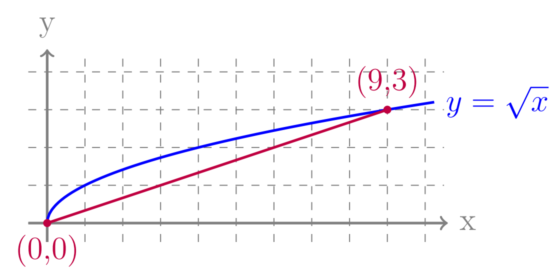
   (i) *Identify and name the quantities in the problem:*

   (ii) *What rate is the problem searching for?*

   (iii) *What are the values of the known variables and rates?*

   (iv) *Which equation(s)* **relates** *the quantities?*

   (v) *Solve.*



2. **Background Story:** Sometimes we can work backward! The expected error in the function is given and you are asked to find the acceptable error in the input measurements.

   Formula to use: $\Delta f \simeq df = f'(a)\Delta x$

   Questions: The standard dime is a cylinder with a volume of $340\,mm^3$ and a height of $1.35\,mm$. In an attempt to scam their local candy store, a teenage counterfeiter has determined that fake coins are accepted if they are within $10\,mm^3$ of the volume of a standard dime. The counterfeiter has total control of height when manufacturing coins.

   (a) What are variable(s), constant(s) and errors of interest of interest in this question? How are the variables related? Which of the variable(s) or error(s) are set? Which ones are you computing?

   (b) What radius should the fake coins have? Round answers to 4 decimal points.

   (c) In order to produce acceptable fake coins, what absolute error is acceptable in the coin's radius? what percentage error is acceptable?

GroupWork Rubrics:

Preparedness: ____/0.5, Contribution: ____/0.5, Correct Answers: ____/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2^{nd} lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

GroupWork Rubrics day 2:

Preparedness: ____/0.5, Contribution: ____/0.5, Correct Answers: ____/0.5

3. **Background Story:** What does it mean to approximate linearly? How do we find the function, the a-value and x?

   Questions:

   (A) *(0.25 points)* Which of the following statements are True?

   (i) Linear approximation means using a linear function to approximate a nonlinear function in a small window of inputs.

   (ii) Linear approximation means using a quadratic function to approximate a linear function in a small window of inputs.

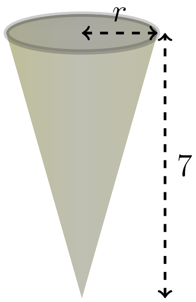
   (B) *(0.5 points)* What linear function approximates a differentiable function $f(x)$ in a small window of input around $x = a$?

   (C) *(2 points)* Use a **linear approximation** to estimate the value of $\sqrt[3]{62}$. (Round to 4 decimal places.)

   (D) *(0.75 points)* Is this an over-estimate or under-estimate of the actual value?

https://youtu.be/PhukhLip4bs and https://youtu.be/7d6nt9KteU8

4. *(3.5 points)* The height of a **conical** vessel is known to be precisely $7\,cm$, and the radius of the base measured as $2\,cm$ with a possible error of $0.2\,cm$. Use **differentials** to estimate the **absolute error** and **percentage error** in computing the **volume** of the vessel.



$$V = \frac{\pi r^2 h}{3}$$

https://youtu.be/kGBXIbfMf9g