Week 11-Lab 1: Worksheet 14: Sections 4.2, 4.3 and 4.4

They said: "How do I study for the exam?" I said: "do each problem on the review, write all details, check your work against the solutions, then put the solutions away and redo the questions."

We said: "We know we have said this before; remember no one learns how to play piano by watching someone play it and no one learns how to play basketball by watching basketball games. Learning require lots of practice and math is no exception. The recommended practice time outside the class for STEM courses is twice to three times of the number of hours in class. Hang in there! Continue to work! Get help from us and other resources on campus. A good foundation in many of those courses can be set for you in Calculus I."

Critical Numbers and Fermat's Theorem

A number c in the domain of f is called a critical number if either $f'(c) = 0$ or $f'(c)$ does not exist.

Fermat's Theorem

If f has a local extremum at $x = c$, and $f'(c)$ exists, then $f'(c) = 0$.

That is, if f has a local max or min at c, then c is a critical number of f.

The Extreme Value Theorem

If f is continuous on a **closed** interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

The Closed Interval Method

To find the absolute extreme values of a continuous function f on a closed interval $[a, b]$:

(1) Find the values of f at the **critical numbers** in (a, b).

(2) Find the values of f at the **endpoints** (namely a and b).

(3) Compare the y-values. The largest value is the absolute maximum value; the smallest value is the absolute minimum value.

First Derivative Test for Local Extrema

Suppose that c is a critical number of a continuous function f.

(I) If f' changes from positive to negative at c, then f has a local maximum at c.

(II) If f' changes from negative to positive at c, then f has a local minimum at c.

(III) If f' does not change sign at c, then c is not a local extremum.

Second Derivative Test for Local Extrema

Suppose that f'' is continuous near c and $f'(c) = 0$.

(I) If $f''(c) < 0$, then $(c, f(c))$ is a local maximum.

(II) If $f''(c) > 0$, then $(c, f(c))$ is a local minimum.

(III) If $f''(c) = 0$, then we cannot draw any conclusion.

Group Work Portion of the Worksheet

Names: ____

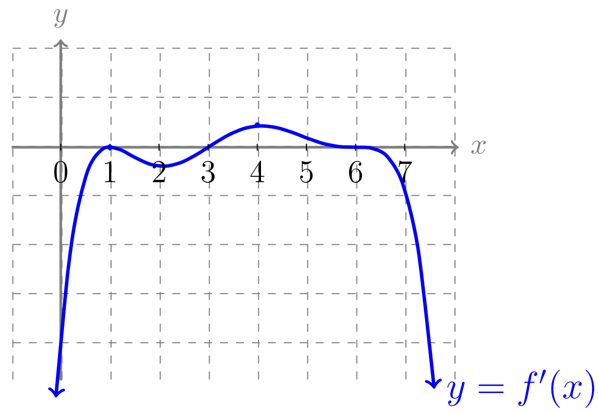
Work in groups to do this portion of the worksheet. Make sure to take parts in solving the problems. Your participation score is a combination of being prepared, willing to explore the problem, working in groups and contributing toward the solution.

1. Consider $f(x) = \sqrt[3]{x}(12 - x)$.

   (a) Find x-value(s) for which f has horizontal tangent line(s). Find x-value(s) for which f has vertical tangent line(s).

   (b) Find the absolute extrema of $f(x) = \sqrt[3]{x}(12 - x)$ on the interval $[0, 12]$.

2. The graph of the **derivative** $f'(x)$ of a continuous function $f(x)$ defined on $[0, 7]$ is shown below. (**Be careful!** This is the graph of $f'(x)$, **not** the graph of $f(x)$.)



(a) On what interval(s) is f increasing?

(b) On what interval(s) is f decreasing?

(c) On what interval(s) is f concave up?

(d) On what interval(s) is f concave down?

(e) At what x-value(s) does f have a local maximum?

(f) At what x-value(s) does f have a local minimum?

https://youtu.be/mqw8Cu-aPg4

3. Using the Mean Value Theorem, if $f(-1) = -2$ and $-3 \leq f'(x) \leq 5$ for all values of x, determine how large and how small $f(4)$ can possibly be.

https://youtu.be/OxzWk9-d0Rc

4. **Background Story:** Absolute value functions have corners when the inside function changes sign. So to find all critical points of a composition of absolute value with an inside function, find all values when the sign of inside function changes and all the critical points of the inside function.

   Questions:

   (A) Solve $x^2 + 3x - 10 > 0$. Solve $x^2 + 3x - 10 < 0$.

   (B) For what values of x, does $f(x) = |x^2 + 3x - 10|$ have corners?

   (C) For what values of f, is $f' = 0$?

   (D) Find the absolute extrema of $f(x) = |x^2 + 3x - 10|$ on the interval $[-6, 0]$.

Optional: Let $f(x)$ be a polynomial. A basic fact of algebra states that c is a root of $f(x)$ if and only if $f(x) = (x - c)g(x)$ for some polynomial $g(x)$. (That is, $x - c$ is a factor of $f(x)$.) We say that c is a multiple root of $f(x)$ if $f(x) = (x - c)^2 h(x)$ where $h(x)$ is a polynomial. (That is, $x - c$ is a factor of multiplicity at least two of $f(x)$.)

(a) Show that c is a multiple root of $f(x)$ if and only if c is a root of both $f(x)$ and $f'(x)$.

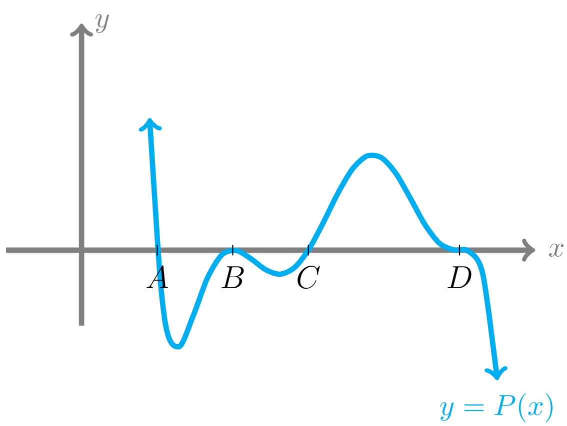
Hints: Verify two statements, "If c is a multiple root of $f(x)$ then c is a root of both $f(x)$ and $f'(x)$" and "If c is a root of both $f(x)$ and $f'(x)$ then c is a multiple root of $f(x)$." I see a product rule in your future in verifying the first statements. Start with what it means for $f(x)$ to have a multiple root the take the derivative and find a factor of $f'(x)$. I also see a product rule in verifying the second statement. This time start with what it means for f to have a root and take the derivative. Then use the fact that c is a root of $f'(x)$ to find a condition on $g(x)$ statement.

(b) Use part (a) to determine whether $c = -1$ is a **multiple** root of each of the following polynomials:

(i) $g(x) = x^5 + 6x^4 + 2x^3 - 20x^2 - 25x - 8$

(ii) $f(x) = x^5 + 6x^4 + 2x^3 - 20x^2 - 27x - 10$

(c) Below is the graph of a polynomial with the roots at A, B, C and D. Which of these are multiple roots? Explain your reasoning using the result of (a).



GroupWork Rubrics:

Preparedness: ___/0.5, Contribution: ___/0.5, Correct Answers: ___/0.5

Individual Portion of the Worksheet

Name: ________________

Upload this section individually on canvas or turn it in to your instructor on the 2^{nd} lab day of the week. You can ask questions in class and work in groups but you turn in the individual work. Start before the class so you can ask questions during the class. If you didn't complete the work in class, make sure to work on it outside the class and complete it. Show all your work; your score depends on the work you have shown.

5. **Background Story:** You need to use this information for one of the questions of the Worksheet 15 individual work so you can graph this function.

   Questions: Let $f(x) = x^{\frac{7}{3}} - 7x^{\frac{4}{3}}$.

   (a) *(1.25 points)* Identify and classify the **local extrema** of $f(x)$ using the **first derivative test** and **second derivative test.**

(b) *(1.25 points)* Find the intervals of **increasing/ decreasing**, **concavity** and **inflection point(s)** of the function, if they exist.

(c) *(1 point)* Find the **absolute extrema** of $f(x)$ on the closed interval $[-1, 8]$.

https://youtu.be/GANwa0UlGtM